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Estimating the Half-life of Advertisements

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Abstract

The effectiveness of an advertisement wears out over time; eventually, it becomes totally ineffective. The author proposes a concept for measuring the lifetime of an advertisement and derives closed-form expressions for it under certain conditions. In addition, the author distinguishes the proposed concept, the half-life of an advertisement, from the prevalent notion of the duration of advertising effects. More importantly, the information on the half-life of ads is actionable from a managerial standpoint, whereas that on the duration of advertising effect is hypothetical. To enable advertisers to estimate the half-life of their ads, the author describes an estimation approach and illustrates its use by applying it to the advertising of the Dockers® brand of Levi Strauss and Company. Substantively, the lifetime of advertisements for the Dockers® brand is about three months. Thus, advertisers are well-advised to periodically estimate the half-life of their ads, so that they can plan the timing to replace worn out advertisements.

Key words: Advertising carryover effects, advertising wearout, aggregate response models, duration of advertising effect, half-life of advertisements, Kalman filter estimation

> "Advertisement is not perceived as enduring work of art, but rather as a vehicle of communication that is doomed to ultimate ineffectiveness as surely as the butterfly is doomed to die."
>
> Weilbacher (1970, p. 219).

1. Introduction

In marketing, it is well known that the effectiveness of advertising wears out over time. The phenomenon of advertising wearout has been studied extensively over the last three decades, from the pioneering field experiments of Grass and Wallace (1969) to the recent modeling developments (e.g., Naik, Mantrala and Sawyer, 1998). For a comprehensive review of this literature, see Pechmann and Stewart (1990). Nonetheless, the lifetime of an advertisement is not known: Is it a few weeks, months, or several years? The empirical knowledge on the lifetimes of advertisements is virtually non-existent because the extant marketing and advertising literatures do not offer concepts and method to estimate it. The objective of this paper is to address this gap in the literature.

Previous research has investigated the carryover effects of advertising (see Assmus, Farley, Lehmann, 1984; Leone, 1995), which relate to the question: how long do advertising effects last (Clarke, 1976)? Although this issue of the duration of advertising effects appears similar to the above issue of the lifetime of advertisements, the two are totally different, conceptually as well as managerially. Conceptually, the duration of an advertising effect is the time required for the *advertising effect* (e.g., brand awareness, goodwill, and sales) to decline to a certain level if *advertising is discontinued forever.* In contrast, a lifetime of an advertisement is the time required for the *effectiveness* of the advertisement to decline to a certain level if *advertising is continued forever.* Managerially, an estimate of the duration of advertising effect provides information to brand managers on how long the current brand awareness level will sustain if the advertising support for the brand is withdrawn. On the other hand, the estimate of the lifetime of an advertisement provides information on when to replace worn-out advertisements if the advertising support is maintained to keep the brand in the mental landscape of consumers. More importantly, I emphasize that the information on the duration of the advertising effect is hypothetical since managers typically do not discontinue advertising for established brands, whereas the information on the lifetime of an advertisement is actionable because managers can plan the timing to replace worn out advertisements (see Pekelman and Sethi, 1978).

In section 2, I formally define and develop the notion of a lifetime of an advertisement, which is inherent in the minds of advertisers (see the above quotation of Weilbacher, 1970), via the concept of a *half-life* of an advertisement. To derive expressions for the half-life of ads, I apply the concept of half-life to the ad wearout model proposed by Naik et al. (1998). To enable advertisers to estimate the half-life of ads for any specific brand, I describe in section 3 an estimation approach based on Kalman filtering methodology (e.g., Harvey, 1994). In section 4, I apply this concept and method to the advertising of Dockers® brand of Levi Strauss and Company. Empirically, I find that the half-life of advertisements for the Dockers® brand is about three months. In contrast, the duration of the advertising effect for the Dockers® brand is as long as three years. Thus, besides the conceptual and managerial differences noted earlier, even the empirical magnitudes of the half-life of ads and the duration of the advertising effect can be quite different. Section 5 concludes by summarizing the contributions of the paper.

2. The Half-life of an Advertisement

In this section, I first define the concept of the half-life of an advertisement. Then, I describe a model of advertising wearout, and derive the analytical expressions for the half-life of ads and for the duration of advertising effect.

2.1 Definition of the Half-life of an Advertisement

A lifetime of an advertisement is infinitely long since the advertiser can decide to telecast it anytime. However, as the quotation in the *Introduction* eloquently describes, the effec-

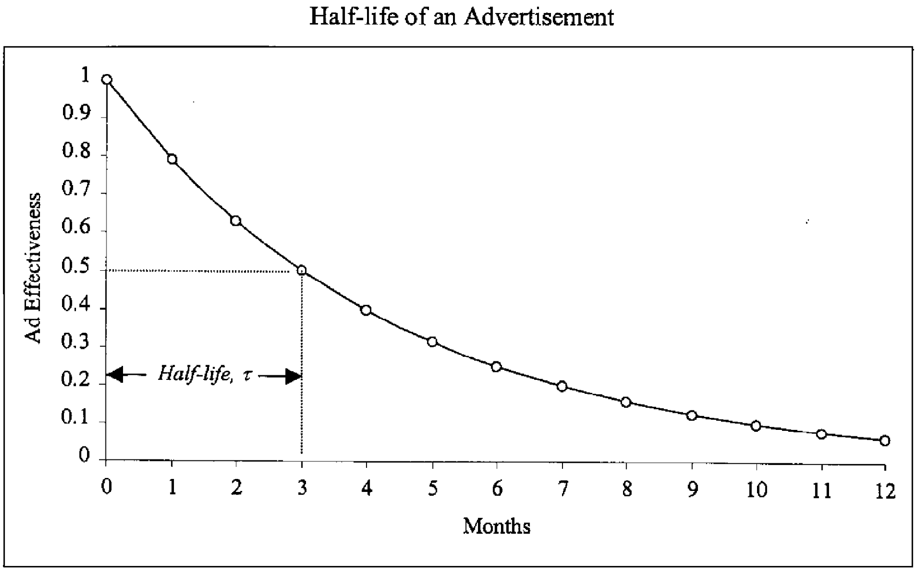

Figure 1.

tiveness of an advertisement wanes, and the advertisement becomes ineffective eventually. Hence, to enable advertisers to plan the replacement of worn-out ads, I propose the following measure for the lifetime of an advertisement:

Definition. The *half-life* of an advertisement is the time required for advertising effectiveness to wear out to one-half of the initial effectiveness level.

Figure 1 illustrates the wear out of ad effectiveness in response to constant advertising over time. The initial ad effectiveness is assumed to be unity and ad effectiveness decays to one-half over a period of three months, which is the half-life of the advertisement (denoted by τ). I next describe a model of advertising wearout so that I can derive the analytical expressions for the half-life of an advertisement.

2.2 Ad Wearout Model

Naik, Mantrala and Sawyer (1998) recently proposed a model of advertising wearout that extends the classical Nerlove and Arrow (1962) model of goodwill formation. Nerlove and Arrow (1962) posit that goodwill for a brand is built by advertising investments made over a period of time, and they model goodwill formation by the following differential equation:

$$\dot{G} = \beta u(t) - \delta G,$$

where $\dot{G}$ denotes an instantaneous change in goodwill for a brand, u(t) is the advertising investment at time t, and parameters β and δ represent ad effectiveness and forgetting rate, respectively. The dot notation in $\dot{G}$ denotes a time-derivative of goodwill G(t); specifically, $\dot{G} = dG/dt$. In Equation (1), the effectiveness of an advertisement, β, is constant over time. Because ad effectiveness changes over time, Naik et al. (1998) extend Equation (1) by modeling a time-varying $\beta = \beta(t, u(t))$. They show that ad effectiveness wears out when advertising is "on" (i.e., $u(t) \neq 0$) and it restores when advertising is "off" (i.e., $u(t) = 0$). Specifically, let I(u) indicate whether advertising is on or off:

$$I(u) = \begin{cases} 1 & \text{if } u \neq 0 \\ 0 & \text{if } u = 0. \end{cases} \tag{2}$$

Then, their model is stated by the differential equation,

$$\dot{\beta} = -a(u)\beta + (1 - I(u))\delta(1 - \beta), \tag{3}$$

where $\dot{\beta} = d\beta/dt$ and a(u) is the rate of ad wearout.

Substituting $I(u) = 1$ in Equation (3), I obtain the wearout of ad effectiveness when advertising is on

$$\dot{\beta} = -a(u)\beta. \tag{4a}$$

Similarly when advertising is off, $I(u) = 0$, the evolution of ad effectiveness is given by

$$\dot{\beta} = -a(0)\beta + \delta(1 - \beta), \tag{4b}$$

and ad effectiveness restores if $\delta(1 - \beta) > a(0)\beta$.

The rate of decline in ad effectiveness, a(u), depends on two sources of wearout: *copy wearout* and *repetition wearout.* Copy wearout, denoted as c, occurs as a result of the passage of time, regardless of the repetitiveness of advertising; that is, $c = a(0)$. In contrast, repetition wearout, denoted as w, is a consequence of the excessive frequency of advertising. Specifically, the parameter w captures the marginal increase in the wearout rate a(u); that is, $w = \partial a/\partial u$. Then, a first-order approximation of a(u) is given by

$$a(u) = c + w\, u. \tag{5}$$

The above discussion completes the description of the model. Equation (1) specifies the formation of goodwill, and the Equations (2) through (5) describe the wearout and restoration of ad effectiveness.

2.3 Determining the Half-life of an Advertisement

To obtain half-life of ads when advertising is on, I integrate both sides of Equation (4a)

$$\int_{t_0}^{t} \frac{d\beta}{\beta} = -\int_{t_0}^{t} a(u)dt, \tag{6a}$$

from any initial time t_0. Substituting Equation (5) in Equation (6a) and integrating, I get

$$\ln\frac{\beta(t)}{\beta_0} = -[c(t - t_0) + w\int_{t_0}^{t} u(t)dt], \tag{6b}$$

where β_0 denotes the initial ad effectiveness. Using the definition of half-life (see subsection 2.1), I first observe that at time $t = \tau + t_0$ the ad effectiveness $\beta(t) = \beta(\tau + t_0) = \beta_0/2$. Then I substitute $\tau = t - t_0$ and $\beta(t)/\beta_0 = 1/2$ in Equation (6b). Hence, I can find the half-life τ by solving the equation:

$$c\tau + w\int_{t_0}^{\tau+t_0} u(t)dt - \ln 2 = 0 \tag{7a}$$

In general, Equation (7a) determines the half life of an advertisement for any advertising schedule u(t). Below I consider two important special cases.

(a) No Repetition Wearout

In the case when repetition wearout is negligible, I substitute $w = 0$ in Equation (7a) and find that the half life of an advertisement is

$$\tau = \frac{\ln 2}{c}. \tag{7b}$$

Equation (7b) provides a simple closed-form expression for the half-life of ads when repetition wearout is negligible and advertising schedule u(t) is arbitrary. Equation (7b) is a "conservative" assessment and provides an *upper bound* on the lifetime of an advertisement because it depends only on the copy wearout parameter c. In other words, if repetition wearout were non-zero, then the half-life τ would be smaller, as shown below.

(b) Constant Advertising

In the case when advertising schedule is constant over time, which is known in the literature as a uniform or even spending schedule (e.g., Mahajan and Muller, 1986), I substitute $u(t) = \bar{u}$ in Equation (7a) and find that the half life of an advertisement is

$$\tau = \frac{\ln 2}{c + w\bar{u}}. \tag{7c}$$

Equation (7c) provides a simple closed-form expression for the half-life of ads under an even spending schedule. Note that the half-life of ads decreases as repetition wearout increases. Next, I derive the duration of advertising effect (Clarke, 1976), which appears similar to—but is different from (see *Introduction*)—the lifetime of ads.

2.4 Duration of Advertising Effect

To obtain the duration of an advertising effect if advertising were discontinued forever, I first set $u(t) = 0$ in the Equations (1), (2) and (3), and find the time required for goodwill to depreciate by 90% of its initial level. Then, the 90% duration of an advertising effect, denoted by $D_{90\%}$, is given by

$$D_{90\%} = \frac{\mathrm{Ln}10}{\delta}. \tag{8}$$

Comparing Equation (8) with the Equations (7b) or (7c), I note that the duration of an advertising effect and the half-life of an advertisement differ not only conceptually and managerially, but also mathematically. Specifically, the duration of an advertising effect does not depend on wearout characteristics of the advertisements (c, w); similarly, the half-life of ads is independent of the advertising carryover effect $(1 - \delta)$.

In the next section I describe an approach to estimate the half-life of advertisements by using commonly available sales-advertising data.

3. Kalman Filter Estimation Approach

I apply the Kalman filter estimation approach (see Naik et al., 1998) to estimate the parameters of model Equations (1), (2), (3) and (5) because it is eminently suited for estimating dynamic models, especially when they involve *coupled* differential equations as in Equations (1) and (3). In this approach, I first obtain the transition equation based on the model dynamics, and then link it to observed sales data to get the observation equation. Using the observation and transition equations, I compute and maximize the log-likelihood function to estimate the model parameters. The details are described below.

Since data are observed at discrete points in time (e.g., weekly, monthly), I discretize Equations (1) and (3) and express them in the following *transition equation*:

$$\begin{bmatrix} G_t \\ \beta_t \end{bmatrix} = \begin{bmatrix} (1-\delta) & u_t \\ 0 & (1-a(u)) - \delta(1-I(u)) \end{bmatrix} \begin{bmatrix} G_{t-1} \\ \beta_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \delta(1-I(u)) \end{bmatrix} + \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix}, \tag{9a}$$

where error terms $[\nu_{1t}, \nu_{2t}]' \sim N(0, \sum_\nu)$, $\sum_\nu = \mathrm{diag}(\sigma^2_{\nu 1}, \sigma^2_{\nu 2})$. These error terms represent the net effects of myriad variables that affect the model dynamics but are not included in the model for the sake of parsimony. The transition equation is expressed compactly as

$$\alpha_t = T_t \alpha_{t-1} + c_t + \nu_t, \tag{9b}$$

where

$$\alpha_t = [G_t, \beta_t]', \; T_t = \begin{bmatrix} (1-\delta) & u_t \\ 0 & (1-a(u)) - \delta(1-I(u)) \end{bmatrix},$$
$$c_t = [0, \delta(1-I(u))]', \text{ and } \nu_t = [\nu_{1t}, \nu_{2t}]'.$$

Next, I link the transition Equation (9a) to observed sales data as

$$S_t = [1 \;\; 0]\begin{bmatrix} G_t \\ \beta_t \end{bmatrix} + \mathbf{X}_t'\gamma + \varepsilon_t, \tag{10a}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Unlike in the awareness response models (e.g., Naik et al., 1998), several factors other than goodwill affect brand sales such as the buying spree during the Christmas season. These factors are included in the covariates $\mathbf{X}_t$ in Equation (10a), which is expressed compactly by the *observation equation*

$$S_t = z'\alpha_t + \mathbf{X}_t'\gamma + \varepsilon_t, \tag{10b}$$

where $z = [1 \;\; 0]'$ is a vector of constants.

To estimate model parameters, I maximize the likelihood of observing sales data $S = (S_1, S_2, \ldots, S_T)'$ for T periods, which is given by

$$L(\Theta; S) = \prod_{t=1}^{T} f(S_t | \Im_{t-1}). \tag{11}$$

The conditional density function $f(\cdot|\cdot)$ of sales S_t, given all information up to the last period $\Im_{t-1}$, is normal. Naik et al. (1998) provide the recursive expressions for mean and variance of $S_t|\Im_{t-1}$ (see their Appendix B). The vector Θ contains model parameters $(c, w, \delta, \gamma, G_0, \beta_0)'$ as well as variances of error terms in the transition and observation equations. Maximizing Equation (11) with respect to Θ, I obtain the maximum likelihood Kalman filter estimates $\hat{\Theta}$.

The difference between this approach and the usual maximum likelihood approach (e.g., Hanssens, Parsons, and Schultz, 1990) is as follows: The latter approach answers the question, "What are the best parameter estimates that maximize the likelihood of observing the data S given the model?" In contrast, the proposed approach solves the following problem: To estimate model parameters, what are the *best transition paths of the unobservable state variables* $[G_t, \beta_t]'$ that maximize the likelihood of observing the data S given the model? That is, the Kalman filter provides the best *state estimates,* $\hat{\alpha}_t = [\hat{G}_t, \hat{\beta}_t]'$ for all t in addition to the best *parameter estimates* $\hat{\Theta}$. Intuitively speaking, the Kalman filter provides the values for the unobserved independent variables $\alpha_t = [G_t, \beta_t]'$ so that the regression model in Equation (10a, b) can be estimated.

The Kalman filter estimates are *optimal*; that is, they are unbiased and have a minimum variance among all estimators when (1) the transition and observation equations are linear in the state variable α_t, and (2) the error terms $[\nu_{1t}, \nu_{2t}, \varepsilon_t]'$ are normally distributed (Harvey, 1994, p. 110). The Equations (9a, b) and (10a, b) satisfy these conditions. Next, I illustrate the use of this approach to estimate the half-life of advertisements for the Dockers® brand.

4. Empirical Half-life of Advertisements

In this section, I first describe the data set and then present the empirical results.

4.1. Data

Dockers® is a leading brand of fashion apparels owned by Levi Strauss & Company. Their ad agency, Foote Cone & Belding, created the *Nice Pants* ad campaign to influence adult men to buy casual Dockers® Khaki Pants. The ad campaign is considered "offbeat" because the advertised product is not shown explicitly (Enrico, 1996). In one of the advertisements, a young man notices a beautiful woman on a subway train; as he tries to reach her, the train pulls away from the platform, but he manages to hear her compliments: "Nice Pants."

Figure 2 shows the pattern of ad spending[1] on network television (see Panel A). Note the intermittent bursts of intense spending levels punctuated by the absence of advertising for four months in each year. This pattern of ad spending is known as *pulsing media schedule* (see, e.g., Mahajan and Muller, 1986). Panel B of Figure 2 shows the retail sales during this period. In Panel B, a sharp increase in sales during months 12, 24, and 36, and a drop in sales in months 13, 25, and 37 are noticeable. To capture these seasonal effects during Christmas, I construct two dummy variables as follows:

$$X_{1t} = \begin{cases} 1 & \text{if } t = 12, 24, \ldots \\ 0 & \text{otherwise,} \end{cases} \qquad X_{2t} = \begin{cases} 1 & \text{if } t = 13, 25, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The covariates defined in Equation (12) are included as regressors in $\mathbf{X}_t = [X_{1t}, X_{2t}]'$ in the observation Equations (10a, b).

4.2 Empirical Results

4.2.1 Parameter Estimates and Model Fit. Table 1 presents the parameter estimates, standard errors, and t-values. These estimates are meaningful from theoretical and managerial viewpoints.

For example, forgetting rate $\hat{\delta} = 0.0637$ is statistically significant and comparable in magnitude to those found in the extant literature (see Assmus, Farley and Lehmann, 1984; Leone, 1995). Copy wearout rate $\hat{c} = 0.2311$ is large and significant, implying that ads become obsolete with the passage of time. However, the repetition wearout rate

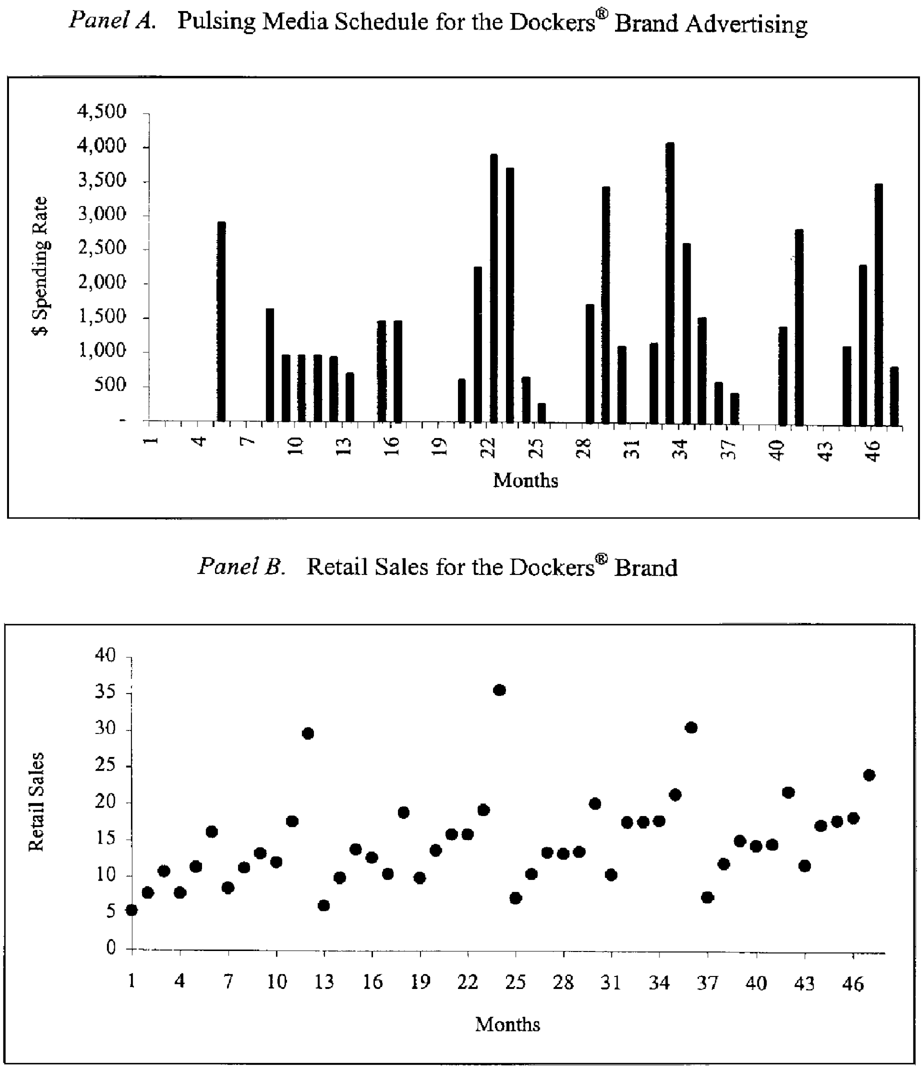

Figure 2.

$\hat{w} = 0.0002$ is small and not significant, suggesting that the frequency of exposure is not too high to bore the target audience. In addition, the seasonal effects during Christmas are significant. The buying spree lifts the retail sales by $\hat{\gamma}_1 = 16.04 \times 100{,}000$ units. In the following month, the drop in sales is estimated at $\hat{\gamma}_2 = 7.44 \times 100{,}000$ units, which may be attributed to product returns or stockpiling by the consumers. Based on $R^2 = 81.17\%$ and Figure 3, the model fit is quite good.

Table 1. Model Parameter Estimates

Parameter	Estimates	Standard Error	t-values
Forgetting rate, δ	0.0637	0.0191	3.34
Copy Wearout, c	0.2311	0.0985	2.35
Repetition Wearout, w	0.0002	0.0077	0.03
Christmas Buying spree, γ_1	16.04	1.6964	9.46
Post-Christmas drop, γ_2	− 7.44	1.4718	− 5.05
Initial Goodwill, G_0	11.26	3.4950	3.22
Initial Ad Quality, β_0	0.043	0.4190	0.10
Model Fit, R^2		81.17%	
Maximized log-likelihood value		− 73.59	

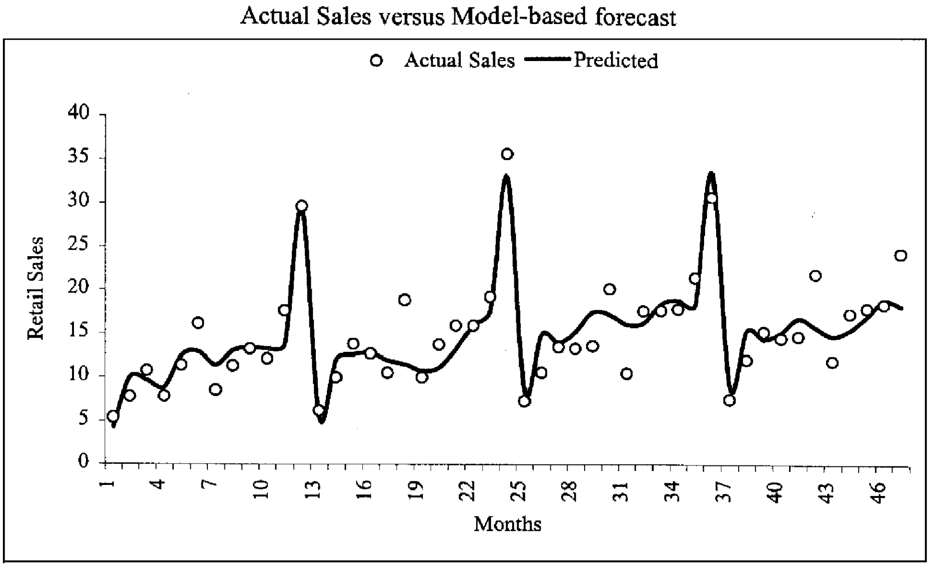

Figure 3.

4.2.2. Empirical Half-life of Ads and the Duration of Advertising Effect. To estimate the half-life of the Dockers® ad campaign, I make use of Equation (7b). This is because I find that the repetition wearout is negligible, since Table 1 indicates that the null hypothesis $H_0 : w = 0$ cannot be rejected at the 5% significance level. *Hence, the estimated half-life of Dockers® ad campaign is* $\hat{\tau} = \mathrm{Ln}2/\hat{c} = 0.693/0.231 \approx$ *3 months.* In contrast, the 90% duration of advertising effect is $\hat{D}_{90\%} = \mathrm{Ln}10/0.0637 \approx 3$ years. Thus, even the empirical magnitudes of the half-life of ads and the duration of the advertising effect are quite different.

5. Conclusion

This paper proposes and develops the concept of the half-life of an advertisement. I define the half-life of ads as the time required for the effectiveness of advertisements to wear out to one-half of the initial effectiveness. I show that the notions of the half-life of ads as proposed in this paper and the duration of advertising effect prevalent in the extant literature are different conceptually, managerially, mathematically and empirically. More importantly, the information on the half-life of ads is actionable from a managerial standpoint because managers can plan to replace worn-out advertisements (see Pekelman and Sethi, 1978). In contrast, the information on the duration of an advertising effect is hypothetical since managers are unlikely to discontinue advertising and observe that the goodwill for their brand actually vanishes after a certain time.

To enable advertisers to estimate the half-life of their ads, I illustrate the application of the Kalman filter approach to the advertising of the Dockers® brand. Substantively, I find that the lifetime of advertisements for the Dockers® brand is about three months. Thus, although they don't last as long as "capital" items such as plant and machinery, advertisements are not as ephemeral as butterflies. By applying the proposed concept and method, advertisers can periodically estimate the lifetime of their ads and appropriately decide the timing to replace worn-out advertisements.

In addition, future researchers can extend this line of inquiry in two ways. Empirically, researchers may shed light on the desirable characteristics (e.g., humor, slice-of-life) to enhance the longevity of ads (see Hanssens and Weitz, 1980). Theoretically, researchers may investigate the issue of designing an optimal compensation scheme based on the half-life of ads, which is an observable assessment of an agency's creative efforts. Such research efforts can potentially improve the practice of advertising.

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Note

1. I re-scale the dollar amounts to maintain confidentiality desired by Levi Strauss and Company.

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