Perils of Using OLS to Estimate Multimedia Communications Effects

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Companies invest millions of dollars in various forms of marketing communications to impact customers' awareness, attitudes, purchases, and, ultimately, profitability. An important question for marketers and shareholders alike is: what effects do marketing investments have on market performance? To assess these effects, marketers estimate marketing-mix models by using regression analysis. However, we show that the estimation of marketing-mix models via regression analysis (i.e., ordinary least squares, OLS) yields severely biased estimates of marketing effects. To mitigate such severe biases, we present an alternative approach, called the Wiener-Kalman filter, that provides reasonable estimates that are much closer to the true parameters than the corresponding OLS estimates. In addition, we analyze Corolla brand's multimedia campaign and furnish results based on marketplace data that corroborate the simulation findings. Finally, we discuss both the implications of these results for brand managers and the opportunities that lie ahead for advertising researchers.

1. INTRODUCTION

Companies invest millions of dollars every year in various forms of marketing communications to influence customers and prospects to buy products and services. For example, General Motors spent over $2.8 billion last year to promote its lines of automobiles. Brand managers, senior management, and shareholders therefore have an interest in knowing whether or not their media advertising had any marketplace effects. Moreover, managers would like to know what combination of media were the most effective to plan future promotional campaigns. To this end, marketing and advertising communities have developed "metrics" that provide market feedback on various measures such as consumer awareness, attitudes, and purchases. Indeed, the metrics related to marketing and marketing communications appear to be the Holy Grail for most marketing managers (see, e.g., Ambler, 2000).

To utilize this information, managers analyze the relevant marketing metrics via techniques that are known as marketing-mix modeling (MMM). Gatignon (1993) and Mantrala (2002) provide a thorough literature review. Marketing-mix models are estimated using regression analysis so that managers can (1) assess the incremental effect of marketing and communication investments in generating sales over that which would normally be expected, and (2) parse out the various media or communication forms that contributed to those increases. Due to increased availability of single-source data from frequent shopper programs, household panels, and other data generating sources, MMM has become the "de facto" tool to determine the effectiveness of marketing activities for major consumer product companies.

The current practice entails tracking weekly brand sales or awareness and then estimating the effects of multiple marketing activities. In the context of marketing communications program, such analyses reveal the effectiveness of multiple media, cross-media synergies, and carryover effects of marketing communication programs. Indeed,

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MMM has become so pervasive in some product categories that few challenge the approach and the resulting output from these models.

The question we raise in this article is whether the practice of estimating marketing-mix models is accurate? With literally billions of dollars of investment decisions resting on the estimation results, perhaps a more complete scrutiny of this ordinary least squares (OLS) methodology is justified.

2. UNCOVERING A SOURCE OF SERIOUS BIASES: MEASUREMENT NOISE

Our central thesis is that the OLS estimates are seriously biased in general, except for special circumstances. Consequently, the OLS approach cannot estimate the marketing-mix models accurately. Why? Because measurement systems that generate the metrics of interest and serve as dependent variables (e.g., awareness, attitudes, and sales) are commonly noisy, imprecise, and fallible.

Standard textbooks would have the manager believe that, while measurement noise in the *independent* variables leads to biased estimates, a noisy dependent variable has no biasing consequence (e.g., see Bollen, 1989, p. 159; Greene, 1993, Chap. 9.5). The received wisdom is that "the measurement error in the dependent vari-

able can be ... ignored" (Greene, 1993, p. 281). However, in the analysis presented in this study, we show that this belief is erroneous when estimating dynamic marketing-mix models. In particular, using both simulated and actual marketplace data, we demonstrate the existence of biases in the OLS estimates. Furthermore, we quantify their magnitudes. Of particular interest to the marketing manager is that, in our Monte Carlo study, the effectiveness of advertising is biased upward; i.e., OLS overstates the impact of media advertising on sales by 34 to 147 percent. In other words, *an estimated effect can be twice as much as what it really is.* As for cross-media synergies and carryover effect, OLS understates their magnitudes by 28 and 48 percent, respectively. These biases are serious because millions of dollars of investments in various forms of marketing communications are based on the output of marketing-mix models estimated via the OLS approach. We substantiate these statements in Section 4.2 and Tables 1 and 2.

Given the perils of OLS estimation, are there alternative approaches a manager can adopt to improve the estimation accuracy? We have found that the Wiener-Kalman filter (WKF) estimation is superior to the OLS approach. To substantiate this claim, in the following sections, we com-

pare the performance of WKF with that of OLS under identical conditions.

We find that WKF yields improved estimates of multimedia campaign effects that are much closer to the true parameters than the corresponding OLS estimates. In addition, it allows the manager to compute a conservative benchmark of the inaccuracy of OLS in real applications. Put differently, because OLS estimates are likely to be inaccurate in practice, the question is whether or not the manager can determine how wrong they are? At least, by knowing the seriousness of the problem, managers will be able to make informed decisions. Finally, we present evidence to demonstrate the existence of OLS biases using actual marketplace data. Specifically, when we analyze the results of the recent Toyota Corolla's multimedia campaign, we find that the OLS estimates of magazine and rebate effectiveness are indeed more than twice as large as they can really be.

The article is organized as follows. Section 3 outlines a model of multimedia communications to quantify the effects. Section 4 reports the simulation results from Monte Carlo experiments, and Section 5 presents the empirical results for Corolla's multimedia campaign. Section 6 discusses the results and related issues, and Section 7 concludes by summarizing the implications for brand managers and the opportunities for advertising researchers.

3. INTEGRATED MARKETING COMMUNICATIONS MODEL

In today's complex marketplace with fragmented media, marketers cannot rely on a single communications medium such as television. Thus they employ multiple media and multiactivity marketing programs. Such programs are often termed as integrated marketing communications (IMC) to distinguish them from the traditional marketing programs that are planned

and implemented separately and independently of each other. Although marketers know that multiple activities together impact consumers' behavior in the marketplace, most marketing-mix models measure the effects of a single activity separately and independently of the other activities. In other words, there is no real integrated planning or measurement model. To substantiate this point, we refer readers to the comprehensive literature reviewed by Feichtinger, Hartl, and Sethi (1994), who conclude that:

With a few exceptions, the [advertising] models assume ... single advertising medium. This was already noted by Sethi (1977), and this critical remark is still valid for the literature published subsequently (Feichtinger, Hartl, and Sethi, 1994, p. 219).

Thus, the marketing-mix models in use today are based on additive models without interaction effects, thereby ignoring the role of cross-media synergies.

To overcome this limitation, marketing-mix models have been extended to capture synergies via interaction effects between multiple activities (see Gatignon and Hanssens, 1987; Gopalakrishna and Chatterjee, 1992; Murthy and Mantrala, 2005; Naik and Raman, 2003; Smith, Gopalakrishna, and Smith, 2004). Next, we describe such an IMC model that quantifies multimedia communication effects.

In traditional marketing programs, the various modes of communication—for example, television, radio, and internet—exert independent effects on consumers in the standard advertising models. Figure 1 depicts this communication process. But managers recognize that consumers combine or integrate the information they receive from various media whether or not the organization itself integrates those
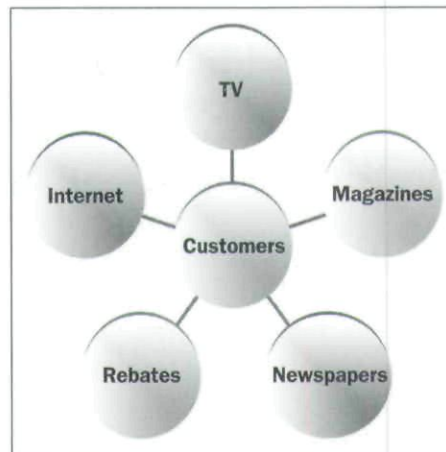

Figure 1 Multimedia Communications Framework

messages across media. The primary challenge then is to conduct marketing activities and send communications messages such that consumers do not integrate them inconsistently and form unintended perceptions about the product or service. Clearly, brand managers should take charge of this integration process by using a *proactive* view of IMC. This perspective marks the emergence of a new approach to media planning (see Schultz and Pilotta, 2004, for details). Figure 2 presents
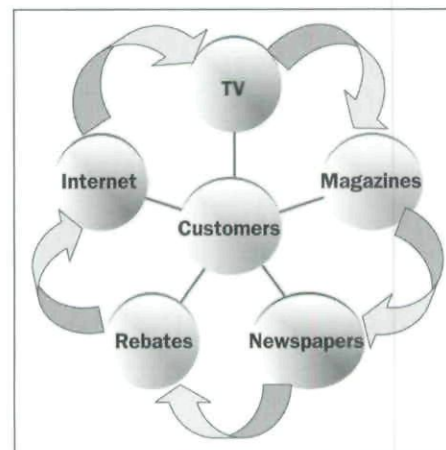

Figure 2 Cross-Media Synergies in IMC

the IMC framework that emphasizes the joint effects or *synergies* of the various marketing activities. These synergies are shown by the curved arrows, generated due to the integration and resulting interactions of multimedia activities.

In comparison to Figure 1, the concept of IMC and integrated media planning is much more than simply using multiple media concurrently as in traditional models, where each activity does *not* depend on any other activity. In contrast, the major difference in the IMC perspective is that the effectiveness of *each* activity depends on *all other* communications activities used by the firm. To understand this distinction, we specify the IMC model developed by Naik and Raman (2003).

3.1. IMC model specification

In their formulation of two media advertising, the growth in brand sales (dS/dt) is driven by the levels of media effort (x_1, x_2), the effectiveness of each medium (β_1, β_2), and the presence of cross-media synergy (κ). The larger the media budget or the greater the media effectiveness or the greater the synergy, the faster the sales growth. Additionally, brand sales would decline at the rate of δ, for example, in the absence of advertising (when $x_i = 0$ for $i = 1, 2$) or attrition to competing brands. Naik and Raman (2003) incorporate these ideas in the continuous-time model, $dS/dt = \beta_1 x_1 + \beta_2 x_2 + \kappa x_1 x_2 - \delta S$, which can be discretized as $\Delta S_t = S_t - S_{t-1} = \beta_1 x_{1t} + \beta_2 x_{2t} + \kappa x_{1t} \times x_{2t} - \delta S_{t-1}$ to obtain the IMC model:

$$S_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \kappa x_{1t} \times x_{2t} + \lambda S_{t-1},$$

$$(1)$$

where the subscript t denotes a specific week (or month) and $\lambda = (1 - \delta)$ represents the carryover effect (e.g., see Clarke, 1976; Leone, 1995).

The major difference in the IMC perspective (from the traditional one) is that the effectiveness of each activity depends on all other communication activities used by the firm.

Rearranging the terms, we get $S_t = \beta_1' x_{1t} + \beta_2' x_{2t} + \lambda S_{t-1}$, where $\beta_1' = \beta_1 + 0.5\kappa x_2$ and $\beta_2' = \beta_2 + 0.5\kappa x_1$. This rearrangement reveals the insight that, due to cross-media synergy, the first medium's effectiveness β_1' depends on the second medium (x_2). Similarly, assuming positive synergies, an investment in the first medium enhances the effectiveness of the second medium (i.e., $\beta_2' = \beta_2 + 0.5\kappa x_1$). Finally, in the absence of synergy (i.e., when $\kappa = 0$), the effectiveness of a medium remains independent of the other medium.

Although Equation (1) employs brand sales, in practice managers can track brand awareness (or attitude change or a number of other marketing metrics) and then specify an IMC model, $A_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \kappa x_{1t} \times x_{2t} + \lambda A_{t-1}$, where A denotes awareness. It is important for managers to recognize that "the mathematics does not care" whether we use sales, attitude change, awareness, or other data in model estimation (Little, 1986, p. 107).

To estimate the multimedia effects $\theta = (\beta_1, \beta_2, \kappa, \lambda)$ in Equation (1), managers conduct regression analysis of sales and advertising data. We describe this regression-based OLS approach in the Appendix. In addition, the Appendix presents an alternative approach, called Wiener-Kalman filter (WKF), to estimate the multimedia effects $\theta = (\beta_1, \beta_2, \kappa, \lambda)$. This alternative approach *explicitly* accounts for the possibility of measurement noise (see Equation (A.3) in the Appendix) to mitigate the estimation biases when marketing metrics are noisy. We note that marketing metrics contain measurement noise because either the process of measurement or the instruments employed (or both) is not perfect. For example, weekly sales for many brands are really "estimates" projected at the national level using sales force or dealer information at the local or regional levels, thereby inheriting a variety of errors of accounting, aggregation, forecasting, recalls, and returns from customers or channel members. As for the metrics such as awareness or attitudes, measurement errors result from not only the measurement process (i.e., phone surveys, web surveys, mall intercepts), but also the instruments employed for collecting information from consumers (e.g., questionnaire design, order of presentation). For ways to reduce these errors, see Bradburn, Sudman, and Wansink (2004).

Indeed, if measurement errors in a dependent variable were innocuous, as standard textbooks suggest (e.g., Bollen, 1989, p. 159; Greene, 1993, p. 281), then we should find that the OLS estimates are comparable to the WKF estimates that explicitly account for the measurement noise. If not, we should expect departure of the OLS estimates from the true parameters, an important issue that we next investigate.

4. MONTE CARLO SIMULATIONS

We conduct three experiments to validate our thesis: measurement noise in the dependent variable induces bias in the OLS estimates of multimedia communications effects. We note that "bias" is defined as the difference between the estimated parameter value and the true parameter value that generated the data. Because the true parameter values are not knowable in real market data, Monte Carlo experiments enable the researchers to generate data under different conditions to determine the magnitude of biases. Three conditions are detailed below: (a) no measurement noise, (b) low measurement noise, and (c) high measurement noise. We first describe the experimental settings and then present the simulation results.

4.1. Experimental settings

We generate the two independent variables by drawing a random sample of size $T = 52$ weeks from $x_i = 50 + U(0, 100)$, $i = 1$ and 2, where $U(0, 100)$ denotes the uniform random variable over the interval $[0, 100]$. We let the media effectiveness $\beta_1 = 1$ and $\beta_2 = 1$, the cross-media synergy $\kappa = 0.01$, and the carryover effect $\lambda = 0.5$. We set $\beta_i = 1.0$ so that the biases in percentages can be computed by eyeballing (e.g., see Tables 1 and 2). We use the central value $\lambda = 0.5$ rather than its extreme values to rule out ceiling or floor effects as confounding factors for the resulting bias. Given that $E[x_i] = 100$, we chose $\kappa = 0.01$ so that the expected contribution of $\kappa x_1 x_2$ in Equation (1) is the same on average as the main effects of $\beta_i x_i$. Would the results obtained from these settings hold for other "realistic" parameter values? We discuss this point in Section 6.6.

The model error term follows the normal distribution with zero mean and standard deviation equal to 50 (i.e., $\varepsilon_t \sim N(0, 50^2)$). Next, starting with the initial sales $S_0 = 100$, we compute the weekly sales S_t for $t = 1, 2, \ldots, T = 52$ by using Equation (A.1) in the Appendix. Finally, we obtain the observed sales Y_t via Equation (A.3), where the observation error

term $v_t \sim N(0, \sigma_v^2)$. In condition (a), $\sigma_v = 0$ because measurement noise is absent. As for conditions (b) and (c), we set $\sigma_v = 50$ and 100 to represent the low and high measurement noises, respectively. For each condition, we create 1,000 data sets, estimate $\hat{\theta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\kappa}, \hat{\lambda})$ via OLS and WKF approaches (see Section 2), and present the averaged estimates in the following results.

4.2. Simulation results

To check the manipulation, we compute that the signal-to-noise ratio, which is defined as the ratio of variances σ_S^2 / σ_v^2, and we find that it decreases from infinity to 5.5 to 1.1 as measurement noise increases from $\sigma_v = 0$ to 50 to 100. Correspondingly, the signal strength, defined as the ratio of variances $\sigma_S^2 / \sigma_Y^2 = \sigma_S^2 / (\sigma_S^2 + \sigma_v^2)$, decreases from 100 percent (excellent) to 84.5 percent (moderate) to 52.4 percent (low).

In Table 1, we report the simulation results for the three experiments. Based on the first experimental condition (a), panel A indicates that the OLS estimates for the multimedia campaign effects—dual media effectiveness, cross-media synergy, and carryover effect—possess negligible departures from the true parameters. Thus, no measurement noise marks a "special circumstance," where the OLS estimates are not biased (unless sample size is small; see Ullah, 2004). Furthermore, the OLS estimates are as good as the WKF estimates, which also are unbiased (asymptotically).

Results of the second experimental condition are given in panel B, which reveals the bias induced in OLS and WKF estimates when measurement noise is low. *The OLS estimates are seriously biased even when measurement noise is low.* Specifically, both the media effectiveness estimates are biased upward by 34 and 41 percent. In contrast, the carryover effect and cross-

TABLE 1

Biases in OLS and WKF Estimates in the Presence of Synergy

Panel A. No Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.02	1.02
Media 2 effectiveness, β_2	1.00	1.01	1.01
Cross-media synergy, κ	0.01	0.0099	0.0099
Carryover effects, λ	0.50	0.4961	0.4967

Panel B. Low Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.41	1.07
Media 2 effectiveness, β_2	1.00	1.34	1.06
Cross-media synergy, κ	0.01	0.0072	0.0097
Carryover effects, λ	0.50	0.4317	0.4877

Panel C. High Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	2.18	1.17
Media 2 effectiveness, β_2	1.00	2.47	1.18
Cross-media synergy, κ	0.01	0.0083	0.0103
Carryover effects, λ	0.50	0.2585	0.4732

media synergy are biased *downward* by 13.6 and 27.5 percent, respectively. Thus, the OLS estimates do not yield accurate estimates of any of the four multimedia effects in the presence of measurement noise in the dependent variable. This finding thus substantiates our central thesis. Furthermore, based on panel B, we infer that measurement errors in the dependent variable are not as innocuous as previously believed (see Greene, 1993, p. 281).

The WKF estimates in panel B are substantially closer to the true parameters than the OLS estimates. The important implication of this finding is that managers should adopt or at least consider the use of the WKF as an alternative ap-

proach to estimate multimedia effects, for real data are likely to possess measurement errors.

Based on the third experiment, panel C shows how biases in the OLS estimates amplify as measurement noise increases. Specifically, both media effectiveness (2.18 and 2.47) are *more than twice as large* as the real effects (1.00). Even the carryover effect and cross-media synergy are severely underestimated by the OLS approach.

It is interesting to observe that *the WKF performs well even when measurement noise is high.* In particular, due to high measurement noise, the signal strength in this condition drops by as much as half from

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100 to 52.4 percent. Yet the resulting biases are only (17, 18, 3, and −5 percent), respectively, for the two media effectiveness, synergy, and the carryover effect.

In sum, we note that the OLS approach is highly sensitive to measurement noise, which drives serious biases in the estimated multimedia effects. It overstates the media effectiveness and understates both the synergy and carryover effects. In contrast, the WKF is robust to measurement noise, yielding parameter estimates that are much closer to the true parameters than those from OLS. We next corroborate these simulation findings with real market data by analyzing Corolla's multimedia campaign.

5. EMPIRICAL ANALYSIS

We acquired the transaction data collected by J. D. Power & Associates on the number of Toyota Corolla cars sold and the promotional rebates offered during October 1996 through June 2002 (see Pauwels, Silva-Risso, Srinivasan, and Hanssens, 2004, for details). For the same period, TNS Media Intelligence provided information on expenditures on various media vehicles such as magazine, television, newspaper, and the internet. Table 2 depicts the descriptive statistics for this sample, indicating that Toyota expends a marketing budget of approximately 10 percent of the dollar sales at retail prices in support of the Corolla brand.

To estimate the IMC model for the Corolla brand, we extend Equation (1) to incorporate effectiveness and synergies for the five marketing activities: manufacturer rebates and advertising via magazines, television, newspapers, and internet. Consistent with Naik and Raman (2003),

we capture the diminishing returns to each advertising expenditures u_t via the square-root transformation, $x_t = \sqrt{u_t}$, relegating the discussion of diminishing returns in Section 6.7. We then estimate this extended IMC model by applying the OLS and WKF approaches described in the Appendix. Because the effectiveness and cross-media synergies for newspaper advertising were statistically insignificant, we exclude that variable from further analysis.

Table 3 reports the OLS and WKF estimates and their corresponding t-ratios (in parentheses). Following our central thesis, to obtain large biases in the OLS estimates, we should establish the presence of measurement noise. To this end, we test the null hypothesis $H_0: \sigma_\nu = 0$, which asserts the absence of noise. We divide the parameter estimate $\hat{\sigma}_\nu = 9.645$ by its corresponding standard error $se(\hat{\sigma}_\nu) = 0.1433$ to compute the t-ratio of 67.29. Because this t-ratio exceeds 1.96, we reject the null hypothesis of no noise and infer that measurement noise level is significant at the 95 percent confidence level (or higher). Thus, there exists large and significant noise in this data sample. Given this inference, we expect biased OLS estimates of multimedia effects and attempt to quantify their magnitudes.

As mentioned earlier, the true parameter values (say, θ_0) are unknowable in real market data. Consequently, the percentage bias in estimated effects, namely, $100 \times (\hat{\theta}_{OLS} - \theta_0)/\theta_0$, cannot be computed. However, our simulations results indicate that the WKF estimates are quite close to the true parameters, and so we compute the relative bias, $(\hat{\theta}_{OLS} - \hat{\theta}_{WKF})/\hat{\theta}_{WKF}$, by replacing θ_0 by $\hat{\theta}_{WKF}$. Table 1 suggests that the relative biases are likely to be smaller than the actual ones, and so this relative bias metric offers a *conservative* benchmark to assess the inaccuracy of OLS in real applications. That is, it

TABLE 2
Descriptive Statistics for Corolla's Multimedia Campaign

Variables	Mean	Standard Deviation
Sales (units per week)	711	211
Magazine advertising ($ per week)	240,803	182,926
Television advertising ($ per week)	421,773	386,745
Internet advertising ($ per week)	2,631	12,721
Newspaper advertising ($ per week)	3,848	12,034
Rebates ($ per week)	177	208
Number of weeks	299	—

TABLE 3
Biases in OLS Estimates for Corolla's Multimedia Campaign

Model Parameter	OLS Estimates	WKF Estimates	Relative Bias in OLS Estimates
Carryover effect, λ	0.7807 (21.55)	0.9351 (42.8)	−16.5%
Media effectiveness			
Magazine advertisements, β_1	0.2479 (2.99)	0.1018 (2.27)	143%
Television advertisements, β_2	0.1817 (2.78)	0.0313 (0.84)	480%
Internet advertisements, β_3	−1.3176 (1.60)	−0.8062 (−0.95)	
Rebates, β_4	0.3077 (2.06)	0.1261 (1.64)	144%
Cross-media synergy			
Magazine × TV, κ_1	−0.0003 (−2.24)	−0.00008 (−1.18)	
Magazine × internet, κ_2	0.0013 (0.86)	0.0006 (0.73)	
Magazine × rebates, κ_3	−0.0005 (−1.11)	−0.0003 (−1.43)	
TV × internet, κ_4	0.0003 (0.32)	0.0003 (0.52)	
TV × rebates, κ_5	0.0000 (−0.46)	0.0000 (0.69)	
Internet × rebates, κ_6	0.0006 (0.15)	0.0006 (0.33)	
Measurement noise, σ_ν		9.6450 (67.29)	
Specification errors, σ_ε		8.6398 (27.59)	

answers the question, how wrong are the OLS estimates in practice?

For Toyota's multimedia campaign for the Corolla brand, we examine the last column of Table 3 to realize that OLS yields *severely* biased estimates in practice, a finding that is consistent with the simulation results (see Table 1). More specifically, OLS underestimates the carryover effect by at least 16.5 percent even in the real data. Similarly, the estimated effectiveness of magazines and rebates are more than twice as large as what they are truly likely to be based on the corresponding WKF estimates. All these empirical results thus corroborate the simulation findings and, together, they lend a strong support for our central thesis.

6. DISCUSSIONS
6.1. What factors drive bias in OLS estimates?

Three factors—lagged dependent variable, small sample size, and measurement noise in the dependent variable—cause OLS estimates to be biased. For instance, Davidson and MacKinnon (2004, pp. 90–91) prove that OLS yields biased estimates for linear dynamic models. Why? Because the next observation in a dynamic model necessarily depends on the previous observation, whereas OLS requires the next observation to be independent of the previous observation. This violation of the independence assumption causes bias in OLS estimates. Consistent with this theory, we find biases in col-

umn 3 of panel A of Table 1, which represents the setting with carryover effects ($\lambda \neq 0$), but no measurement noise (i.e., $\sigma_\nu = 0$). In addition, Ullah (2004, Chap. 6) explains that OLS estimates are biased in small samples (see his Tables 6.1 and 6.2, pp. 140–141 for the extent of biases). It is important to recognize that the magnitude of these biases are small—about 1 to 2 percent in column 3 of panel A—and do not create major concerns to practicing advertisers and marketers.

In contrast, measurement noise in the dependent variable is a major source of bias. Panel B of Table 1 furnishes strong evidence that measurement noise induces biases in OLS estimates over and beyond those due to the lagged dependent variable. Specifically, by comparing panels A and B, we learn that OLS estimates are *substantially* biased—from 14 to 41 percent in columns 2 and 3 of panel B—due to *noisy* dependent variable. Furthermore, if measurement noise drives bias, then its magnitude should increase as measurement noise increases. Substantiating this point, panel C reveals that OLS biases increase as measurement noise increases (compare the corresponding entries in the third column of panels B and C). Thus, measurement noise in the dependent variable—and not so much the lagged dependent variable per se—causes large biases in OLS estimates.

To gain intuition for the bias due to measurement errors, consider the graph in Figure 3 for the regression of y on the single x-variable. The bold line depicts the true slope; the dashed line displays the estimated slope. The measurement errors in the x-variable inject *additional* variability, as shown by the curved arrows, thereby perturbing the estimated slope downward systematically. This phenomenon manifests itself for *each* of the multiple x-variables, but the direction of bias becomes complicated; it can be upward
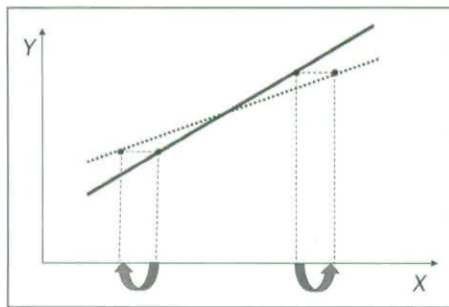
Figure 3 Graphical Intuition for the Biased Slope Estimate due to Measurement Errors

TABLE 4

Biases in OLS and WKF Estimates in the Absence of Synergy

Panel A. No Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.02	1.02
Media 2 effectiveness, β_2	1.00	1.02	1.02
Carryover effects, λ	0.50	0.4892	0.4892

Panel B. Low Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.14	1.04
Media 2 effectiveness, β_2	1.00	1.19	1.02
Carryover effects, λ	0.50	0.4111	0.4838

Panel C. High Measurement Noise

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.4541052	1.09
Media 2 effectiveness, β_2	1.00	1.4271299	1.06
Carryover effects, λ	0.50	0.2660	0.4577

or downward (e.g., see Carroll, Ruppert, and Stefanski, 1995, for further details). Furthermore, as we have shown in this article, errors in the dependent variable are *not* innocuous in dynamic models—as they would be in static models—because the lagged dependent variable serves as a regressor.

6.2. The role of synergy

What happens if no synergy exists? Is OLS still biased? To gain insights into the role of synergy, we re-do all the simulation studies assuming no synergy. Specifically, we set synergy $\kappa = 0$ in Equation (1) and keep all the other settings in Section 3.1 unchanged to allow for direct comparisons with previous results. Table 4 displays the new results. Comparing the corresponding entries in Tables 1 and 4, we conclude that (1) biases in OLS estimates are substantially larger in the presence of synergy than in its absence, and (2) the proposed WKF estimates are not nearly as biased in finite samples (and are unbiased asymptotically).

Consistent with these findings, OLS estimation seemingly detects significant negative synergy (t-value = −2.24) between Corolla's magazine and television advertising. However, this finding is likely to be misleading because we know from the reported simulation studies that the OLS estimates of synergy are biased *downward*. In other words, OLS could suggest the presence of negative synergy even when this may not be the case. Indeed, based on the corresponding WKF estimate, we infer the absence of synergy (t-value = 1.18). Thus, the biases in OLS estimation not only lead to inaccurate magnitudes, but also incorrect inferences.

6.3. Bayesian estimation and shrinkage estimators

Although the proposed procedure, the WKF, finds its home in modern control theory and engineering literatures, it is equivalent to Bayesian estimation. Specifically, Meinhold and Singpurwala (1983) provide limpid introduction to Kalman filtering from a nonengineers' perspective and without using control-theoretic language, explaining its equivalence to Bayes' theorem. Briefly, the filter discounts noisy observed data proportional to the magnitude of noise, thereby "shrinking" it toward the prior means, which are often based on advertising theory (such as Equation (1)).

Other shrinkage estimators developed recently include Wavelets Shrinkage (Donoho and Johnstone, 1994), Lasso (Tibshirani, 1996), or Constrained Inverse Regression (Naik and Tsai, 2005). Intuitively, shrinkage estimators tend to pull the estimates toward zero; so upwardly biased parameters "shrink" toward true values, but underestimated parameters can move further away from their true values.

More specifically, applying wavelet shrinkage to advertising awareness data, Naik and Tsai (2000) proposed a Denoised

Least Squares (DLS) estimator to control for measurement noise. Their Monte Carlo studies compare various estimators; the main results show that DLS performs quite well, and it outperforms OLS, but *not* the Kalman filter on statistical measures (e.g., mean squared error) and managerial metrics (e.g., budget, profit). In other words, a shrinkage estimator helps or hurts depending on the *direction* of bias; hence further investigation is necessary.

6.4. Nonlinear dynamic advertising models

Observed dependent variables in advertising models consist of variables other than sales, such as advertising awareness or market shares, which lie between 0 to 100 percent. Such boundaries can *potentially* introduce nonlinear effects. Even so, the linear Kalman filter works, provided the dependent variable is not "too close" to either zero or unity. For example, applying the linear Kalman filter to advertising awareness data, Naik, Mantrala, and Sawyer (1998) estimate copy wearout and repetition wearout for improving media planning. Applying the linear Kalman filter to market share data, Naik, Raman, and Winer (2005) discover negative synergy in advertising and promotion activities and show how managers should optimally allocate marketing dollars in oligopoly markets in the presence of (neg-

ative) synergy. However, if observed data are close to zero or unity, then an appropriate nonlinear Kalman filter needs to be designed. To design nonlinear filters, researchers should apply the theory of extended Kalman filter or "particle filters" based on sequential Monte Carlo method (which is not used yet in advertising and marketing literatures).

6.5. What if some firms use methods other than OLS?

We demonstrate biases in OLS estimation because (1) standard textbooks suggest that errors in the dependent variable can be ignored (see Bollen, 1989, p. 159; Greene, 1993, p. 281), which is evidently incorrect for dynamic advertising models (see Tables 1, 4, and 5), and (2) the results can be *verified* via simple simulations. This latter reason forms the foundation of any scientific discipline. If consulting firms use other methods, they should disclose full details so that the extent of bias resulting from those methods can be assessed independently by applying simulation settings developed in this article. The lack of transparency for proprietary methods hinders verification of the properties of the resulting estimates—but that does not mean absence of biases. As for clients, we recommend that they should demand (and pay for) reports to ensure unbiasedness of advertising effectiveness, synergy, and

carryover effects—for multimillion dollars of marketing budget can be saved.

6.6. Realistic parameter values

To conduct simulations, we have to choose some specific parameter values. A natural question arises, would those results generalize to other parameter values? Fortunately, the answer is both affirmative and verifiable. That is, the phenomena revealed by these simulations hold for other feasible parameter values (which are infinitely many)—namely, $\beta_i \in (0, \infty)$, $i = 1$, 2, $\lambda \in (0, 1)$, and $\kappa \in (-\infty, \infty)$— and not just the specific values chosen for these experiments. To illustrate this point, we change the carryover effect from 0.5 to 0.9, which is a more "realistic" value (e.g., see Table 3). Then we conduct 1,000 additional simulations for the low measurement noise condition, holding constant all other settings in Section 4.1. Finally, Table 5 presents new results, which are qualitatively equivalent to those reported in Tables 1 and 4.

6.7. Role of diminishing returns in budgeting decisions

We captured diminishing returns using "square root" transformation of dollars spent. This assumption corroborates with the notion of "convex costs" prevalent in operations research, economics, and marketing. However, as an anonymous reviewer points out, how should advertisers ascertain that the square-root transformation, and not some other function, is the best nonlinear response function for their product markets? Furthermore, this shape could vary across different media: the bang for doubling radio advertising need not be the same as that for doubling internet advertising. If so, how does optimal budgeting and allocation of resources change with respect to varying *degree* of diminishing returns for different media? These issues are underresearched in extant

TABLE 5

Biases in OLS and WKF Estimates with Realistic Carryover Effect

Advertising Effects	True Parameters	OLS Estimates	WKF Estimates
Media 1 effectiveness, β_1	1.00	1.13	1.025
Media 2 effectiveness, β_2	1.00	1.13	1.028
Cross-media synergy, κ	0.01	0.0094	0.00999
Carryover effects, λ	0.90	0.8946	0.8985

advertising and marketing literatures. Hence we encourage future researchers to generate answers to these open questions.

7. CONCLUSIONS

Standard regression theory suggests that a noisy dependent variable is innocuous because "measurement error ... can be absorbed in the disturbance of the regression and ignored" (Greene, 1993, p. 281). Our central thesis, contrary to this belief, is that measurement noise has substantial biasing effects in dynamic markets. Consequently, brand managers, advertising agencies, market research consultants, advertising researchers, and marketing scientists should be cautious when using regression-based estimates of media effectiveness, cross-media synergy, and carryover effects for substantive decision making. *Biases in OLS estimates can be as high as 100 percent*—both simulation and empirical results support this finding (see Tables 1, 3, and 4).

Given these results, we recommend that regression analysis be routinely supplemented with the WKF estimates, so that managers learn the magnitudes of biases in the OLS estimates before making budgeting and allocation decisions. A stronger form of this recommendation is that the WKF approach should be used, instead of the OLS approach, to estimate dynamic multimedia models because the WKF estimates are closer to the truth than the OLS estimates. Indeed, users can decide which approach they should adopt. We simply provide evidence and tools for the manager to judge for himself or herself the extent of biases resulting from these methods. We next elucidate the implications of such biases for budgeting and allocation.

7.1. Implications for brand managers

We have shown that the commonly used regression analyses overstate the advertising effectiveness and understate the carryover and synergy effects (see Tables 1, 3, and 4). These biases affect the managerial decisions related to the level of budget for IMC programs and its allocation across several years. In the short run, managers relying on the OLS estimates are likely to overspend on advertising because advertising appears to be more effective than it truly is. In the long run, however, managers would commit to a smaller marketing budget than they should due to the underestimation of the carryover effect (which captures the long-term effectiveness of IMC programs). In other words, the mistake managers could make would be to misallocate a greater proportion of the marketing budget to short-run activities relative to long-term brand-building activities. Furthermore, the underestimation of cross-media synergy implies that managers would incorrectly allocate a smaller budget than necessary for achieving media integration. Overall, these implications suggest that present resource allocation by brand managers may tend to be myopic because of the inaccuracy of the OLS approach.

7.2. Opportunities for advertising researchers

In this article, we have shown that (1) the commonly used regression analysis yields substantially biased estimates of the effects of multimedia communications; (2) a serious source of these biases is measurement noise; (3) the WKF, when used, mitigates those biases even in the presence of measurement noise. Besides measurement noise, advertising researchers can further investigate the impact of other important factors. Specifically, the nature and magnitude of biases caused by nonstationarity of parameters are unknown. For example, advertising effectiveness can vary over time due to various factors— for example, wear-in and wearout of advertising (see Blair, 2000; Naik, Mantrala, and Sawyer, 1998). So a research question could be: How well does OLS, relative to WKF, estimate such time-varying parameters? Similarly, we need a better understanding of estimating dynamic multimedia models with unobservable states, for example, those introduced by periodic economic recessions. Recently, Smith, Naik, and Tsai (2006) explored how two discrete states of the economy— recessions and expansions—can *differentially* affect advertising effectiveness and carryover effects. But their model ignored the role of multiple media and, to augment its usefulness, it needs to be extended to include cross-media synergies.

In conclusion, the media marketplace has exploded in the past few years. Managers are faced with an almost unlimited choice of media and marketing communication alternatives. The traditional method of allocating and measuring each investment separately and independently ignores the reality of today's media marketplace. Although regression analysis is commonly used for estimating dynamic marketing-mix models, it is seriously challenged in terms of the accuracy of estimated effects. Given the perils of using regression analysis (or OLS estimation), we need better approaches to obtain more accurate estimates of marketplace effects. We believe the use of the WKF approach, introduced in this article, is a relevant next step in the development of IMC models going forward. Such efforts would advance the theory and improve the practice of integrated multimedia communications. **JAR**

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APPENDIX

Description of OLS and WKF Estimation Approaches

OLS ESTIMATION OF THE IMC MODEL

In the regression analysis, we introduce the error term ε_t in Equation (1) to obtain

$$S_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \kappa x_{1t} \times x_{2t} + \lambda S_{t-1} + \varepsilon_t, \quad (A.1)$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Conceptually, the error term represents the *specification errors*, which capture the role of myriad other factors that are not explicitly included in the model for the sake of par-

simony. Next, for each week $t = 1, 2, \ldots, T$, we stack the sales observations S_t in the vector $S = (S_1, S_2, \ldots, S_T)'$ and the media observations $X_t = (x_{1t}, x_{2t}, z_t, S_{t-1})'$ in the matrix $X = (X_1, X_2, \ldots, X_T)$, where $z_t = x_{1t} \times x_{2t}$. Then, using the sample from $t = 2, \ldots, T$ (because we do not have the initial observation for the lagged sales), we obtain the ordinary least squares (OLS) estimates via the formula,

$$\hat{\theta}_{OLS} = (XX')^{-1} XS. \quad (A.2)$$

WKF ESTIMATION OF THE IMC MODEL

In Wiener-Kalman filter (WKF) theory, we acknowledge not only the specification errors ε_t in Equation (A.1), but also the possibility that the observed data could be noisy and imprecise because measurement systems generating them are fallible (i.e., not perfect). To incorporate the presence of measurement errors, we specify an observation equation,

$$Y_t = S_t + \nu_t, \quad (A.3)$$

where $\nu_t \sim N(0, \sigma_\nu^2)$. If $\sigma_\nu = 0$, then measurement noise is absent (as in OLS). If not, we obtain an estimate of the level of measurement noise in the data. In addition, as we shall illustrate in the empirical example, managers can test the null hypothesis $H_0: \sigma_\nu = 0$ to determine whether or not measurement noise is significant based on market data (rather than speculate data quality based on own belief).

Next, we use Equations (A.1) and (A.3) to compute the log-likelihood of observing the sales trajectory $Y = (Y_1, Y_2, \ldots, Y_T)'$, which is given by

$$LL(\Phi; Y, X) = \sum_{t=1}^{T} g(Y_t \mid \Im_{t-1}), \quad (A.4)$$

where $g(\cdot \mid \cdot)$ is the conditional density of sales Y_t given the history up to the last period, $\Im_{t-1}$. The random variable $Y_t \mid \Im_{t-1}$ is normally distributed for all t, and its mean and variance are given recursively by the Kalman filter (see Naik, Mantrala, and Sawyer, 1998, p. 234). Because both the model and noise parameters are stationary, the Kalman filter is identical to the filter derived by Wiener (1949), and hence we call it the Wiener-Kalman filter.

Then, we stack the model parameters $(\beta_1, \beta_2, \kappa, \lambda)'$ as well as the variances of specification and observation errors and the initial sales in a hyper-parameter vec-

tor Φ. By maximizing the likelihood function in Equation (A.4), we obtain the WKF estimates,

$$\hat{\Phi}_{WKF} = \text{ArgMax} \, LL(\Phi \mid Y, X). \quad (A.5)$$

The standard errors of these estimates are obtained from the information matrix evaluated at estimated parameter values. The resulting WKF estimates are asymptotically unbiased and possess minimum variance among *all* possible estimators because Equations (A.1) and (A.3) are linear in S_t and the error terms $(\nu_t, \varepsilon_t)'$ are normally distributed (Harvey, 1994, p. 110).