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Few studies address the marketing budgeting problems of platform firms operating in two-sided markets with cross-market network effects, such that demand from one customer group in the platform influences the demand from the other customer group. Yet such firms (e.g., newspapers whose customers are both subscribers and advertisers) are prevalent in the marketplace and invest significantly in marketing. To enable such firms to make effective marketing decisions, the authors delineate the desired features of a platform firm's marketing response model, specify a new response model, and validate it using market data from a local newspaper. The results show that the firm faces reinforcing cross-market effects, its demand from both groups depends on marketing investments, and the model exhibits good forecasting capability. The authors use the estimated response model to determine optimal marketing investments over a finite planning horizon and find that the firm should significantly increase its newsroom and sales force investments. With this model-based recommendation, the firm's management increased its newsroom budget by 18%. Further normative analysis sheds light on how cross-market and carryover effects alter classical one-sided marketing budgeting rules.

Keywords: marketing budgeting, marketing dynamics, Kalman filter, two-sided markets

Dynamic Marketing Budgeting for Platform Firms: Theory, Evidence, and Application

A fundamental responsibility of marketing managers is to determine the optimal levels and allocation of scarce marketing resources. Consequently, a large volume of work in marketing models literature has focused on developing normative rules for marketing resource allocation decisions (e.g., Ingene and Parry 1995; Raman 2006), investigat-

ing the effectiveness of firms' marketing efforts empirically (e.g., Hanssens and Ouyang 2002; Manchanda and Honka 2005), and building implementable model-based tools for optimizing marketing investment decisions in specific settings (e.g., Pauwels, Arts, and Wiesel 2010). However, surveys of this literature (Hanssens, Parsons, and Schultz 2001; Mantrala 2002; Shankar 2008) reveal that research to date has largely ignored marketing budgeting and allocation decisions by a substantial segment of firms in the economy, namely, platform firms that do business in two-sided markets (e.g., Evans 2003).

Platform firm markets are distinguished from one-sided firm markets, in that they have two or more different groups of customers (i.e., end users of their products or service offerings) that the businesses aim to acquire and retain (Rochet and Tirole 2005). Examples include print media such as newspapers and magazines (readers and advertisers), television broadcasting (viewers and advertisers), shopping malls (shoppers and retailers), payment cards (cardholders and merchants), and sports clubs (spectators and sponsors). Specifically, platform firms' two-sided markets are characterized by (1) the existence of two or more distinct groups of customers interested in different offerings

of the platform, (2) at least one customer group whose main interest is gaining access to the other group, and (3) the need for the platform to facilitate such access more efficiently than would bilateral relationships between the members of the groups (Evans 2003). Thus, a platform firm's demand from one customer group depends on the demand for it from the other customer group, such that platform firms face cross-market network effects (CME) (e.g., Chen and Xie 2007). Intuitively, when CMEs are present, a platform firm's marketing efforts to stimulate demand from one customer group can have repercussions for its other customer group.

Hereafter, we refer to the end-user group that is primarily interested in and consumes an offering of the platform itself, regardless of the presence or absence of any other end-user group, as *attractors* (e.g., consumers of a newspaper's editorial content, or readers). We refer to the end-user group interested in accessing attractors through the platform as *suitors* (e.g., advertisers buying space in a newspaper for their advertisements to readers). Figure 1 provides a diagrammatic representation of a newspaper firm allocating marketing efforts to its attractors and suitors. For example, a newspaper invests in enhancing its product quality (newsworthy content) to retain and increase the number of readers. At the same time, it invests in a sales force to promote sales of ad space to suitors. An increase in the number of suitors can affect attractors' future demand for the newspaper. Specifically, an increase in the ratio of advertising to editorial content in the newspaper potentially increases or decreases demand from attractors. (Advertising has a positive effect on readers when the ad provides information deemed valuable by the readers, but some readers are ad averse [e.g., Sonnac 2000].) Thus, these two sources of revenue—readers' purchases of the newspaper's editorial content and advertisers' purchases of the newspaper's ad space—are interrelated.
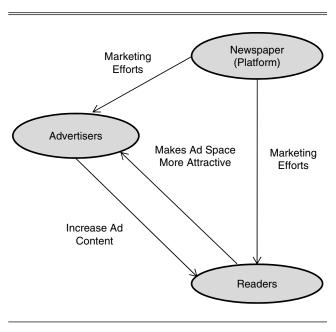
Although many such firms exist, including *Fortune* 100 companies like Time Inc. (magazine) and FOX (television network), literature on optimal marketing resource allocations by platform firms is scarce. Two novel and challenging aspects of these firms' marketing budgeting decision problems that any model-based solution must address are (1) the differential dynamic (e.g., carryover) effects of marketing on the demands from the dual or multiple sides of the platform firm's business and (2) the CMEs between the multiple end-user groups. Evans and Schmalensee (2007, p. 12) effectively summarize these challenges: "its [the platform's] customer groups form a dynamic system and live in a non-linear world.... Changes in customers of one type affect customers of the other type," so the firm "must consider the interdependence of these two groups of customers at every turn."

Motivated by these observations, we propose a model and develop a theory for optimal marketing investment decisions by platform firms. Specifically, the objectives of our research are threefold:

1. Propose a platform firm market response model, taking into account the quintessential features of the two-sided firm setting, and validate this proposed response model using market data from an archetypal platform firm, namely, a daily print newspaper company.
2. Develop a model-based algorithm and decision tool to assist managers in determining the optimal paths (trajectories), over some finite planning horizon, of marketing investments toward the two sides of their market, and demonstrate the application and benefits of this tool for the participating newspaper company.
3. Derive new normative results for platform firm marketing budgeting to extend current one-sided marketing budgeting theory.

To meet the first objective, we propose a dynamic response model that captures the key features of a platform firm's two-sided market setting: CMEs, dynamic effects, and interaction between the demand of one end-user group and the marketing effort directed at the other group. Then, using data from the local daily newspaper firm, we estimate the proposed response model using state-space methods (e.g., Naik, Mantrala, and Sawyer 1998; Xie et al. 1997). Three important findings result from this empirical analysis. First, the data support the proposed two-sided market response model, such that it performs better than a model without CMEs. Second, the estimated attractor and suitor effects are both significantly positive, which indicates that this newspaper is a reinforcing platform, in contrast with Wilbur's (2008) empirical finding of ad-averse television viewers. Third, the significant CMEs imply marketing efforts have both direct and indirect effects, such that efforts toward one end-user group influence the other end-user group. Furthermore, we replicate the proposed response model using data from a different newspaper, held by a different company, and obtained similar results. This replication study enhances our confidence in the validity of the proposed market response model.

To meet the second objective, we develop an algorithm to determine the investment trajectories (time paths) that maximize the discounted long-term profits of the focal platform firm over a specified planning horizon. Because of the finite horizon nature of the problem and the complex

Figure 1
NEWSPAPER ALLOCATING MARKETING EFFORTS

interdependent demand dynamics, we encounter a nonlinear, two-point boundary value problem. We solve this problem by extending the approach proposed by Naik, Raman, and Winer (2005) to our platform firm setting. We incorporate this algorithm in a model-based decision aid that managers can use to evaluate and compare the outcomes of any selected investment-mix trajectories with those of the optimal policies. Thus, we contribute to reducing the "acute shortage of normative studies developing navigation systems that allow managers to optimize marketing efforts" (Leeflang et al. 2009, p. 16).

Subsequently, we demonstrate the practical benefits of this decision aid for the focal newspaper firm. Specifically, its marketing investments toward readers and advertisers were suboptimal. Managers were underspending on newsroom resources and investments in news quality that would generate revenues from attractors (readers). By reallocating resources to achieve the optimal levels, they could increase profits by approximately 28%. From our recommendations, the company's management was not only persuaded of the value of a resource allocation based on econometric and optimization methods but also decided to increase the firm's newsroom budget by approximately $500,000 (an 18% increase in the current newsroom budget). This decision stands in stark contrast with the recent cuts in newsroom investments made by other U.S. daily newspapers to improve their financial performance (e.g., Rosentiel and Mitchell 2004). Thus, we believe that other newspapers also would benefit from the proposed model and decision aid for making informed marketing decisions.

Finally, to meet our third objective, we perform a normative analysis and deduce three new propositions that shed light on how dynamic CMEs and carryover effects modify the traditional rules for optimal marketing investments in the absence of CMEs. For example, we establish that compared with one-sided settings, optimal investment levels should be higher (lower) for a reinforcing (counteractive) platform firm for which both CMEs are positive (the two CMEs have opposite signs).

The rest of this article is organized as follows: The next section reviews literature on platform firms that reveals the gap in research on optimal marketing budgeting by such firms. We then summarize how the platform firm's setting differs from the one-sided firm's setting, delineating the relevant features to capture the institutional reality of the two-sided setting. Next, we specify and validate the proposed response model empirically. Subsequently, we develop the marketing mix algorithm and demonstrate its application to aid decision making. Finally, the normative analyses yield new propositions on optimal marketing investments by platform firms. We close by summarizing the key takeaways and identifying avenues for model extensions and further research.

PLATFORM FIRM BUSINESS STRATEGIES: SELECTED LITERATURE REVIEW

Platform firms cater to two (or more) distinct groups of customers, with members of at least one group wishing to access the other group (Evans 2003). Economists have primarily focused on platform firm pricing strategies. For example, Parker and Van Alstyne (2005), Rochet and Tirole (2005), Armstrong and Wright (2007), and Bolt

and Tieman (2006) examine how standard pricing policies should be restructured in the presence of two-sidedness, and Caillaud and Jullien (2003) study how pricing rules should change in a setting of competing platforms. Roson (2004) provides a detailed review of pricing-related work on two-sided markets. His review highlights that when cross-market network effects (CMEs) are present, (1) prices applied to the two market sides are both directly proportional to the price elasticity of the corresponding demand (Rochet and Tirole 2005); (2) socially optimal pricing in two-sided markets leads to an inherent cost recovery problem, inducing losses for the monopoly platform (Bolt and Tieman 2006); and (3) in a duopoly, the platform charging the lower fees could capture both sides of the market and result in a market monopoly (Caillaud and Jullien 2003). Eisenmann, Parker, and Alstyne (2006) summarize strategic pricing management takeaways from the preceding economics literature. They note that many emerging platform firms struggle to establish and sustain their two-sided networks because of a common mistake: "In creating (pricing) strategies for two-sided markets, managers have typically relied on assumptions and paradigms that apply to products without network effects. As a result, they have made many decisions that are wholly inappropriate for the economics of their industries" (Eisenmann, Parker, and Alstyne 2006, p. 3).

Marketing literature on platform firms is small but growing. For example, Chen and Xie (2007) examine the relationship between high levels of attractor loyalty and platform firm profits under competition. Wilbur (2008) estimates a structural model of suitor (advertiser) demand for attractors (viewers) and viewer demand for advertisers in the television industry and finds evidence for ad aversion among viewers. Gupta, Mela, and Vidal Sanz (2009) develop a model to calculate the customer lifetime value of the buyers (attractors) and sellers (suitors) in an auction house and find that the buyer's value is higher than that of the seller. Kind, Nilssen, and Sørgard (2009) investigate how competitive forces may influence the way media firms such as television channels and newspaper firms raise revenue—from advertisers, through direct payments from attractors, or both. They show that that the less differentiated the media firms' content, the larger is the fraction of their revenue coming from advertising. In contrast, direct payment from media consumers becomes more important as the number of competing media products increases.

The marketing mix problem of the platform firm has received limited attention in prior literature. A notable exception is work by Mantrala et al. (2007), who employ a static model to study the optimality of marketing expenditures toward subscribers and advertisers of daily newspaper firms. However, their model specification meets only a few of the requirements for an ongoing platform firm, as we delineate in the next section. Moreover, we perform a longitudinal analysis, develop an optimal allocation algorithm, and demonstrate its use to assist a firm's managers plan dynamic marketing mix investment policies over finite planning horizons.

Next, we present the key elements of platform firm market response models and how they differ from a one-sided ("classic") firm's response model and allocation problems studied in extant literature (e.g., Mantrala 2002). We focus on monopoly models, consistent with the empirical setting of local daily newspapers.

ONE-SIDED VERSUS TWO-SIDED FIRMS' MARKETING RESPONSE MODELS

A monopoly firm's marketing budgeting and planning problems can be addressed by formulating a market response model that relates demand from one revenue source (e.g., product, region, end-customer group) to one or more marketing variables (e.g., advertising, sales force). Furthermore, we distinguish between a firm's marketing budgeting and allocation problems that involve only a single sales entity (e.g., single product) versus multiple sales entities (e.g., the multiproduct problem treated by Doyle and Saunders 1990). Finally, some optimal marketing budgeting analyses are static (e.g., Dorfman and Steiner 1954), whereas others are dynamic with sales decay or carry-over effects (e.g., Nerlove and Arrow 1962). Thus, classic monopoly firm budgeting problems are of the following types: static or dynamic, single sales entity problems (see Table 1 for market response model types I and II) and static or dynamic, multiple sales entity problems (see Table 1 for model types III and IV). Notably, in the classic model types III and IV, a marketing input set for one sales entity (e.g., advertising, price for one product) could have a cross-price (e.g., Reibstein and Gatignon 1984) or a "spillover" effect on demand for the other product (e.g., Erdem and Sun 2002; Ingene and Parry 1995). Thus, the marketing effort, say advertising, aimed at one market segment (e.g., consumers in Germany) can directly affect (through a spillover effect) another segment (e.g., consumers in Belgium; see Brody and Finkelberg 1997; Gensch and Welam 1973).

As we noted, all platform firms cater to at least two end-user groups or sales entities, making them inherently multiple sales entity problems. How do they differ from model types III and IV? First, all platform firm problems involve two or more distinct end-user (customer) groups as sales entities, each with its own budget constraint and seeking primarily a different offering from the firm. In contrast, classic firms' marketing budgeting decision problems that involve multiple sales entities need not involve distinct customer groups. For example, a problem involving spending on promotions of complementary products (e.g., cake mix and frosting) pertains to the same end users. Of course, classic firm problems sometimes involve two or more distinct end-user groups (with distinct budgets), as in Brody and Finkelberg (1997). We distinguish such multigroup classic firm problems from those of platform firms: The main difference is that, for a classic firm, the level of demand from one group does not directly affect the demand from the other group. In contrast, in a platform firm problem, the level of demand from one group directly affects the demand from the other group and vice versa, such that cross-network effects arise for platform firms but not for the classic firm. Model types V and VI in Table 1 indicate this distinction in the market response models for platform firms. Moreover, because of these network effects, one group's demand increases as a function of the other's demand when the network effect is positive (i.e., reinforcing platform). When one of the network effects is negative, the platform is counteractive. Also, the marketing effort toward one group and the level of the demand of the other side may have an interactive effect on the demand from the first group. For example, the effect of selling effort aimed at a newspaper's advertisers is likely to vary with the number of its subscribers. Third, in multigroup classic firm problems, the demand from either group remains positive even if the demand from the other side goes to zero. For example, demand for a product in Germany exists even if the demand for it in Belgium is zero (and vice versa). However, in a platform firm setting, the demand from at least one side, the suitors (e.g., advertisers), vanishes if attractors' (e.g., readers') demand for the platform disappears. Table 1 indicates this distinguishing feature of platform firms. To summarize, the essential features of a response model for a platform firm's marketing are as follows:

1. The demand from one side should directly affect the other side's demand for the relevant offering of the platform (and vice versa).

Table 1

CLASSIC (ONE-SIDED) VERSUS PLATFORM FIRM MARKETING BUDGETING AND RESPONSE MODELS

Model Type	Description	Specification	Example
I	Static single entity	$\text{Sales}_t = f(\text{Mktg}_t)$.	Dorfman and Steiner (1954)
II	Dynamic single entity	$\text{Sales}_t = f(\text{Sales}_{t-1}, \text{Mktg}_t)$.	Nerlove and Arrow (1962)
III	Static multiple entity	$\text{Sales}_{1t} = f(\text{Mktg}_{1t}, \text{Mktg}_{2t})$. $\text{Sales}_{2t} = f(\text{Mktg}_{1t}, \text{Mktg}_{2t})$. $\partial(\text{Sales}_{it})/\partial(\text{Sales}_{jt}) = 0, \, i \neq j$.	Ingene and Parry (1995)
IV	Dynamic multiple entity	$\text{Sales}_{1t} = f(\text{Sales}_{1t-1}, \text{Mktg}_{1t}, \text{Mktg}_{2t})$. $\text{Sales}_{2t} = f(\text{Sales}_{2t-1}, \text{Mktg}_{1t}, \text{Mktg}_{2t})$. $\partial(\text{Sales}_{it})/\partial(\text{Sales}_{jt-1}) = 0, \, i \neq j$.	Gensch and Welam (1973)
V	Static platform firm	$\text{Sales}_{1t} = f(\text{Sales}_{2t}, \text{Mktg}_{1t})$. $\text{Sales}_{2t} = f(\text{Sales}_{1t}, \text{Mktg}_{2t})$. when $\text{Sales}_{it} = 0$, $\text{Sales}_{jt} = 0, \, i \neq j$ for at least one side. $\partial(\text{Sales}_{it})/\partial(\text{Sales}_{i,t-1}) = 0, \, i, j$.	Mantrala et al. (2007): (they do not allow $\text{Sales}_{it} = 0 \Leftrightarrow \text{Sales}_{jt} = 0, \, i \neq j$, for at least one side)
VI	Dynamic platform firm	$\text{Sales}_{1t} = f(\text{Sales}_{1t-1}, \text{Sales}_{2t-1}, \text{Mktg}_{1t})$. $\text{Sales}_{2t} = f(\text{Sales}_{1t-1}, \text{Sales}_{2t-1}, \text{Mktg}_{2t})$. when $\text{Sales}_{it} = 0$, $\text{Sales}_{jt} = 0, \, i \neq j$, for at least one side. $\partial(\text{Sales}_{it})/\partial(\text{Sales}_{j,t-1}) \neq 0, \, i \neq j$	This article

2. At least one side's (attractors') demand should have a positive direct effect on the other side's (suitors') demand level.
3. The suitors' demand should be zero when the attractors' demand is zero.
4. Attractors' demand should remain nonzero and finite even when suitors' demand is zero.
5. An interactive effect between marketing to one side and demand from the other should be allowed.
6. One side's demand should be a monotonic function of the other side's demand.

In addition, for optimal budgeting by a platform firm, desirable model features are as follows:

7. Previous period demand should affect current demand on the same side.
8. Demands should increase at a decreasing rate as contemporaneous marketing efforts increase.

In the next section, we propose a platform response model with these features. Note that Mantrala et al. (2007) do not incorporate features 3 or 5–7. Thus, by generalizing their static model, we deepen understanding of planning marketing investments by platform firms in the presence of dynamic cross-market network effects.

MODEL DEVELOPMENT

A Two-Sided Market Response Function

We consider a monopolist platform firm such as a local daily newspaper (98% of daily newspapers are the only ones published in their market; Picard 1993). We focus on how the sales on both sides of the platform grow in response to marketing communications investments (e.g., Naik, Prasad, and Sethi 2008; Simon and Arndt 1980) rather than price, because prices stay fixed for long durations. For example, newspaper retail prices remained constant over four to seven years (Bils and Klenow 2002), and advertising rates for local newspapers remain unchanged for a year after they are set (Warner and Buchman 1991).

Let A_t and S_t denote the dollar sales revenues at time t from the attractor and suitor sides of the market, respectively. Also, let u_t and v_t represent the marketing investments of the platform toward its attractor and suitor sides, respectively. Then we specify the platform's dynamic sales marketing effort response system as follows:

$$(1) \qquad A_t = A_{t-1}^{\lambda_A} \tilde{S}_{t-1}^{\theta_{SA}} u_t^{\beta_u} e^{\kappa_1 \varphi_t}, \text{ and}$$

$$(2) \qquad S_t = \tilde{S}_{t-1}^{\lambda_S} A_{t-1}^{\theta_{AS}} v_t^{\beta_v} e^{\kappa_2 \phi_t},$$

where $\tilde{S}_{t-1} = \text{Max}\{S_{t-1}, \varphi_t\}$ and $\varphi_t = \{1 \text{ when } S_{t-1} = 0, \text{ and } 0 \text{ when } S_{t-1} > 0\}$.

Equation 1 states that sales from the attractor side is a product of attractor sales in the previous period (A_{t-1}), suitor sales in the previous period (S_{t-1}), and contemporaneous attractor market-focused marketing investment (u_t), such as investments in news quality. The exponent of A_{t-1} (i.e., λ_A) represents the attractor dynamic effect; the exponent of u_t (i.e., β_u) represents the attractor marketing sales elasticity. Specifically, we capture diminishing returns to the current-period marketing investment when $0 < \beta_u < 1$. Next, θ_{SA} denotes the cross-market effect (CME) of suitor sales in the previous period on current attractor sales (or suitor repercussion effect). This parameter value can be positive or negative, depending on whether attractors value suitors' use of the platform or not; for example,

newspaper readers may be ad lovers (Sonnac 2000), or television viewers may be ad averse (Wilbur 2008).

Similarly, in Equation 2, the exponents λ_S, β_v, and θ_{AS} represent, respectively, the suitor dynamic effect, direct effect of current investment v_t, and dynamic effect of attractor demand on suitor demand. Diminishing returns to marketing spending in Equations 1 and 2 rule out unbounded growth beyond a certain sales level, because incremental marketing dollars do not draw additional readers (or advertisers) profitably, leading to finite optimal spending. We expect the CME θ_{AS} to be positive, because suitors seek access to attractors, and their demand for the medium of the platform should increase when they observe a higher level of attractors' demand for the platform. Thus, Equations 1 and 2 together represent the market response model for the platform firm setting, which is reinforcing when $\theta_{AS} > 0$ and $\theta_{SA} > 0$ and counteractive when $\theta_{AS} > 0$ and $\theta_{SA} < 0$.

The complete specifications of Equations 1 and 2 allow attractor sales to remain nonzero and finite when $S_{t-1} = 0$. That is, the model allows the platform to increase its attractor base even if it intermittently obtains zero revenues from suitors. For example, some media platforms may not secure nonzero advertiser revenues every period. Specifically,

• When S_{t-1} is nonzero, attractor demand in period t depends on the cross-market effect of suitor demand in period $t-1$, as well as level of attractor demand in $t-1$ and investments in marketing toward attractors in period t. Suitor demand in period t depends on the cross-market effect of attractor demand in period $t-1$, as well as level of suitor demand in $t-1$ and investments in marketing toward suitors in period t. That is,

$$(\tilde{1}) \qquad A_t = A_{t-1}^{\lambda_A} S_{t-1}^{\theta_{SA}} u_t^{\beta_u}, \text{ and}$$

$$(\tilde{2}) \qquad S_t = S_{t-1}^{\lambda_S} A_{t-1}^{\theta_{AS}} v_t^{\beta_v}$$

(because φ_t is 0, e^0 is 1, and $\tilde{S}_{t-1} = S_{t-1}$).
• When S_{t-1} is zero, attractor demand in period t depends only on the level of attractor demand in the previous period and investments in marketing toward attractors in period t. Suitor demand in period t arises only because of the cross-market network effect of attractor demand in $t-1$ and investments in marketing toward suitors in period t: $A_t = A_{t-1}^{\lambda_A} u_t^{\beta_u} e^{\kappa_1}$ and $S_t = A_{t-1}^{\theta_{AS}} v_t^{\beta_v} e^{\kappa_2}$ (because φ_t is 1, $\tilde{S}_{t-1} = 1$, and $1^{\theta_{SA}}$ is 1; κ_1 and κ_2 are freely estimable intercepts).

In summary, the general system of Equations 1 and 2, constituting the platform firm response model, possesses the desirable features:

• The parameters θ_{AS} and θ_{SA} allow for direct demand interdependence.
• With a positive θ_{AS}, suitors' demand increases as attractors' demand increases and equals zero when attractor sales vanish, $S_t = 0$ when $A_{t-1} = 0$.
• Attractor demand remains nonzero and finite even when suitors' demand is zero.
• The multiplicative form of Equations 1 and 2 implicitly incorporates the interactions between marketing investment toward one side of the market and demand from the other side of the market. That is, the total effect of a contemporaneous marketing effort toward one side of the market augments the accumulated sales on the other side.

The system of Equations 1 and 2 poses novel estimation and optimization challenges. On the estimation side, considering their interdependent nature, the platform firm's demands are affected by correlated shocks over

time. Therefore, ordinary least squares–based estimates are biased, because they ignore intertemporal dependence (Naik and Tsai 2000). On the optimization side, to obtain dynamically optimal marketing budgets with the response system of Equations 1 and 2, we need to solve a multivariate nonlinear boundary value problem, with direct interdependence between the states (revenues) and controls (marketing investments), a problem that is new to marketing literature. We explicate how we resolved these estimation and optimization challenges in the next two sections. In our empirical application, the case of $S_{t-1} = 0$ does not arise for any t (though the proposed estimation and optimization approaches hold for the general system).

EMPIRICAL ANALYSIS

We present the data from a major newspaper company, describe an estimation approach, conduct model selection and diagnostics, and furnish the empirical results. In addition, we replicate the findings by using a different newspaper's data to lend further validity to the proposed model.

Data

A privately held media company, which has diversified holdings in newspaper and magazine publishing and wishes to remain anonymous, provided the data for its major newspaper. Medium-sized newspapers with fewer than 85,000 subscriptions form its core business. The particular print newspaper we examined is a monopolist in its city-region, producing differentiated content with local features. A third-party audit bureau verified the subscription figures and provided demographic information (e.g., age, gender, income, home ownership) on the newspaper's readers (attractors) to its prospective advertisers (suitors), who may purchase ad space in the future. The newspaper appeals mainly to advertisers who seek to reach audiences older than 50 years, including financial companies and assisted living centers. Because the newspaper invests heavily in marketing to these advertisers, its share of local advertisers' print advertising budgets is quite high.

The data set contains information on revenues from attractors (readers) and suitors (advertisers). In addition, the monthly marketing efforts toward these two revenue sources, namely, dollar spending on newsroom and ad space sales force, were available. Prior work in journalism literature suggests that investments in the newsroom are akin to investments in product quality (Litman and Bridges 1986), as the newsroom department is responsible for providing accurate and engaging news stories to its diverse local readers. Newsroom investments vary with (1) the hiring or termination of part-time employees in the newsroom, (2) changes in the population and demographics trends in the county, (3) changes in the amount of retailer-based economic activity in the county, and (4) the occurrence of important events in the county (e.g., local government elections). The field sales force's main task is to provide recent figures about the size and composition of the attractor base to suitors, as well as inform them about the potential benefits of purchasing ad space in certain sections of the newspaper that their targeted attractors might read. As we show in Table 2, the newspaper spends about equally in the newsroom and on the sales force. Panels A and B in Figure 2 contain plots of the attractor and suitor sales over

Table 2
DESCRIPTIVE STATISTICS

Variable[a]	M	SD
Attractor revenues (subscription)	60.04	4.43
Suitor revenues (advertising)	202.4	19.45
Newsroom investments	22.14	1.30
Sales force investments	21.02	2.56

[a]All variables are in US$10,000 per month.

time. We observe from Panel B in Figure 2 that the special case of suitor sales equal to zero for some t does not apply in our empirical setting. Panel C in Figure 2 contains the plot of the marketing investments over time.

Estimation Approach

We apply filtering theory (e.g., Harvey 1994; Jazwinski 1970) to calibrate the proposed model using market data. Specifically, our response model represents a system of difference equations with nonlinear decision variables, intertemporal dependence of demand, and potentially correlated error structures. In practice, the observed data on attractor and suitor sales may contain measurement errors. Naik and Tsai (2000) show the importance of separating the dynamics of market response from measurement errors when estimating market response functions. Therefore, using the Kalman filter (KF), we develop an algorithm with three steps: (1) the transition equation step that specifies the sales dynamics, (2) the observation equation step that links the sales dynamics to actual sales data, and (3) a likelihood function built recursively and subsequently maximized to obtain the parameter estimates and infer statistical significance. We describe the three steps in turn. (If suitor sales equal zero, we would augment the variable space with the definitions of $(\tilde{S}_{t-1}, \varphi_t)$ and extend the parameter vector to include κ_1 and κ_2.)

Step 1. The transition equation specifies the model dynamics and captures the influence of marketing efforts. We log-transform the response model to get

$$(3) \quad \begin{bmatrix} Ln(A_t) \\ Ln(S_t) \end{bmatrix} = \begin{bmatrix} \lambda_A & \theta_{SA} \\ \theta_{AS} & \lambda_S \end{bmatrix} \begin{bmatrix} Ln(A_{t-1}) \\ Ln(S_{t-1}) \end{bmatrix} + \begin{bmatrix} \beta_u Ln(u_t) \\ \beta_v Ln(v_t) \end{bmatrix}.$$
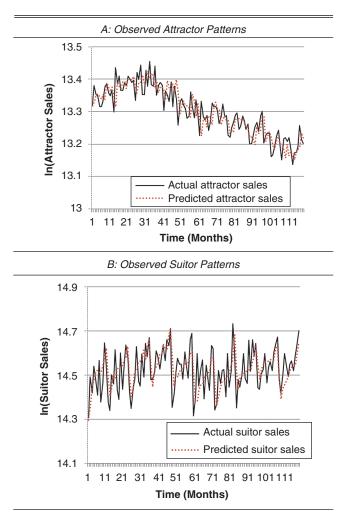
Denoting $Z_{At} = Ln(A_t)$, $Z_{St} = Ln(S_t)$, $w_{ut} = Ln(u_t)$, and $w_{vt} = Ln(v_t)$, we specify the transition equation as

$$(4) \quad \begin{bmatrix} Z_{At} \\ Z_{St} \end{bmatrix} = \begin{bmatrix} \lambda_A & \theta_{SA} \\ \theta_{AS} & \lambda_S \end{bmatrix} \begin{bmatrix} Z_{A,t-1} \\ Z_{S,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{u,t} \\ \beta_v w_{v,t} \end{bmatrix} + \begin{bmatrix} \vartheta_{A,t} \\ \vartheta_{S,t} \end{bmatrix},$$
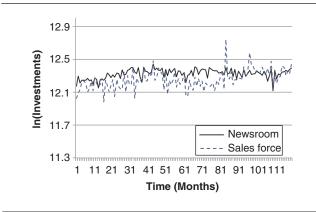
where A_t and S_t represent the attractor and suitor revenue, respectively; $w_{u,t}$ and $w_{v,t}$ represent log-transformed investments toward the attractors and suitors, respectively; and β_u and β_v represent marketing effectiveness. The transition error vector $\vartheta_t = (\vartheta_{A,t}, \vartheta_{S,t})'$ follows $N(0, Q)$, where

Q is the 2×2 covariance matrix, which can be nondiagonal to allow for correlated shocks in the system. The initial means of the transition vector $Z_0 = (Z_{A,0}, Z_{S,0})'$, which are analogous to regression intercepts, are estimated from the market data.

Attractor sales exhibit a downward trend (see Figure 2), consistent with the general decline in print newspaper readership (e.g., Patterson 2007). Suitor sales exhibit sea-

Figure 2
SALES AND INVESTMENT GRAPHS

A: Observed Attractor Patterns

B: Observed Suitor Patterns

C: Newsroom/Sales Force Investments

sonality during the year. To incorporate these aspects, we augment the transition equations with a time-trend variable for the attractor sales dynamics and seasonal dummies for the suitor sales dynamics. The augmented transition equation is

$$
(5) \quad
\begin{bmatrix} Z_{At} \\ Z_{St} \end{bmatrix}
=
\begin{bmatrix} \lambda_A & \theta_{SA} \\ \theta_{AS} & \lambda_S \end{bmatrix}
\begin{bmatrix} Z_{A,t-1} \\ Z_{S,t-1} \end{bmatrix}
+
\begin{bmatrix} \beta_u w_{u,t} \\ \beta_v w_{v,t} \end{bmatrix}
+
\begin{bmatrix} \gamma_1 t \\ \gamma_2 D_{1,t} + \gamma_3 D_{2,t} \end{bmatrix}
+
\begin{bmatrix} \vartheta_{A,t} \\ \vartheta_{S,t} \end{bmatrix},
$$

where γ_1 captures the trend effects on Z_{At}, whereas γ_2 and γ_3 control for the seasonal year-end and beginning effects with the dummy variables D_{1t} and D_{2t}, defined as follows:

$$
(6) \quad
D_{1t} =
\begin{cases}
1, & \text{if } t = (11, 12), (23, 24), \ldots, \\
 & \quad (119, 120) \\
0 & \text{otherwise,}
\end{cases}
$$

and

$$
(7) \quad
D_{2t} =
\begin{cases}
1, & \text{if } t = (1, 2), (13, 14), \ldots, \\
 & \quad (109, 110) \\
0 & \text{otherwise.}
\end{cases}
$$

Step 2. We link the transition equation to observed data as follows:

$$
(8) \quad
\begin{bmatrix} Y_{At} \\ Y_{St} \end{bmatrix}
=
\begin{bmatrix} Z_{At} \\ Z_{St} \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{S,t} \end{bmatrix},
$$

where Y_{At} and Y_{St} represent the actual log-transformed observed values of attractor and suitor revenues, and the observation error vector $\varepsilon_t = (\varepsilon_{A,t}, \varepsilon_{S,t})'$ follows $N(0, H)$, where H represents a 2×2 matrix that can be nondiagonal to allow for correlated shocks to the system.

Step 3. Using the KF recursions (e.g., Harvey 1994) and denoting $Y_t = (Y_{At}, Y_{St})'$, we compute the log-likelihood function,

$$
(9) \quad
LL(\Psi) = \sum_{t=1}^{T} Ln[p(Y_t \mid \Im_{t-1})],
$$

where $p(\cdot \mid \cdot)$ denotes the conditional density of Y_t, given the history of information up to the previous period, $\Im_{t-1} = \{Y_1, \ldots, Y_{t-1}\}$. The vector Ψ contains the model parameters $(\lambda_A, \lambda_S, \theta_{AS}, \theta_{SA}, \beta_u, \beta_v, \gamma_1, \gamma_2, \gamma_3)'$, together with the observation and transition covariance matrices and the initial means. By maximizing Equation 9 with respect to Ψ, we obtain the parameter estimates $\hat{\Psi} = \arg\max(LL(\Psi))$ and infer their statistical significance using the information matrix.

Model Selection and Diagnostics

Applying the preceding approach to newspaper data, we estimated Equations 5 and 8, which include trend and seasonality, dynamic effects, marketing effectiveness, CMEs, and correlated errors. To determine the variables to be retained in the model, we apply model selection theory (see Burnham and Anderson 2002). The central idea of model

Table 3
MODEL SELECTION

Numbers	Models	AIC	AIC$_c$	BIC	MAD (Fit) (%)	MAD (Forecast) (%)
1	Trend, seasonality, and lagged effects only	−771	−767	−739	.61	.63
2	Trend, seasonality, lagged effects, marketing variables, no CMEs	−1,035	−1,030	−998	.40	.42
3	Trend, seasonality, lagged effects, marketing variables, CMEs, uncorrelated errors	−1,120	−1,115	−1,080	.39	.43
4	Trend, seasonality, lagged effects, marketing variables, CMEs, correlated errors	−1,122	−1,116	−1,079	.26	.35

Notes: AIC = Akaike information criterion, AIC$_c$ = corrected Akaike information criterion, BIC = Bayesian information criterion, MAD = mean absolute deviation, and CME = cross-market network effects.

selection is to balance parsimony (i.e., include few variables) and fidelity (i.e., improve goodness of fit). By including additional variables in the model, we can improve the model's fit to the observed data but at the cost of overparameterizing the model, which reduces estimation precision and forecasting accuracy. Information criteria such as the Akaike information criterion (AIC), bias-corrected Akaike information criterion, and Bayesian information criterion are commonly used to compare nested or nonnested models. A smaller value of an information criterion indicates a better model. In addition to the information criteria, we conduct diagnostic checks by computing the mean absolute deviation (MAD) of the model's predicted attractor and suitor outcomes. In addition, we test for the exogeneity of newsroom and sales force investments by applying the approach recommended by Engle, Hendry, and Richard (1983) and provide the details in Appendix A.

We compare the four nested models that incrementally introduce the phenomena of interest. Model 1 includes trend, seasonality, and carryover effects; Model 2 adds marketing variables to Model 1; Model 3 introduces CMEs to Model 2; and Model 4 admits correlated error terms to Model 3.

In Table 3 we report the results of the model selection. First, Model 2 with marketing effectiveness variables outperforms the one with lagged effects, trend, and seasonality variables only (Model 1); the AIC value improves by 34.2%, from −771 to −1035. Second, Model 3 with CMEs outperforms both Model 1 and Model 2; the AIC value (−1120) improves by 45.2% over Model 1 and 8% over Model 2. Third, Model 4 with CMEs and correlated errors outperforms the other three models; the AIC value (−1122) improves by 45.5% over Model 1, 8.4% over Model 2, and .2% over Model 3. Therefore, we retain Model 4 for further analyses.

The retained model with CMEs and correlated errors also fits the data well; Table 3 shows low in-sample MAD (.26%). Furthermore, to assess predictive ability, we estimate the models using 90 observations and forecast the remaining 30 observations in the holdout sample using the calibrated model. We find that its predictive ability is also high, as evidenced by low out-of-sample MAD (.35%). In Figure 2, we present the visual evidence for the proximity of actual versus estimated sales. Finally, Engle, Hendry, and Richard's (1983) test for exogeneity shows that newsroom and sales force investments are weakly exogenous (for details, see Appendix A). Thus, the model selection and

diagnostic checks furnish evidence that the proposed model is a parsimonious specification, fits the data well, and forecasts satisfactorily. We now present the empirical results.

Empirical Results

Control variables. In Table 4 we present the key parameter estimates and t-values. A significant value of $\gamma_1 (−.001, p < .05)$ indicates a declining trend in attractor revenues. The significant estimates $\gamma_2 (.04, p < .05)$ and $\gamma_3 (−.11, p < .05)$ suggest seasonality in suitor revenues. Specifically, we find a statistically significant increase in suitor revenue in the Thanksgiving and Christmas season, followed by a drop-off at the beginning of the year. This finding comports with the experience of many small newspapers in the United States; for example, the *Monroe County Advocate* designs a "Christmas Carol" supplement to accommodate more ad space during holiday months, because approximately 41% of news readers find ads most helpful during shopping sales (Newspaper Association of America 2006).

Cross-market effects. We find that the attraction effect and the suitor effect are both positive and significant $(\theta_{AS} = .16, p < .05; \theta_{SA} = .12, p < .05)$, suggesting that this particular newspaper is a reinforcing platform. Positive suitor effects suggest that unlike television viewers who are ad averse (Wilbur 2008), newspaper readers value advertising. Several factors support this finding: Newspapers are

Table 4
ESTIMATION RESULTS

Parameter	Estimate	t-Value
Trend in attractor revenue (γ_1)	−.001	−14.35
Year-end ad revenue rise (γ_2)	.04	2.43
Year-beginning ad revenue drop (γ_3)	−.11	−12.06
Attractor revenue intercept (α_{A0})	13.27	7.88
Suitor revenue intercept (α_{S0})	14.51	11.38
Attractor revenue carryover (λ_A)	.69	26.05
Suitor revenue carryover (λ_S)	.63	26.56
Attractor cross-market effect (θ_{AS})	.16	6.74
Suitor repercussion cross-market effect (θ_{SA})	.12	2.26
Effectiveness of attractor-directed marketing (β_u)	.25	7.60
Effectiveness of Suitor-directed marketing (β_v)	.18	8.63

a high-attention medium not suitable for multitasking, the newspaper ads are "keepable" because they can be cut out and used at a later period, and newspapers are viewed as a less intrusive and more trustworthy source of information (Conaghan 2006).

Sales dynamic effects. Because the estimated values are less than 1 in magnitude, both parameters represent sales carryover effects, and the attractor carryover and suitor carryover coefficients are positive and significant ($\lambda_A = .69$, $p < .05$; $\lambda_S = .63$, $p < .05$). A moderate value of $\lambda_A = .69$ suggests that newly acquired attractors may not stay with the newspaper for extended periods of time. This finding corroborates with the general trend of declining readership and the idea that local readers may not find enough community content in the newspaper (Project for Excellence in Journalism 2008).

Marketing effectiveness. The effectiveness of newsroom investments with respect to attractor revenues and the sales force with respect to suitor revenues are both positive and significant ($\beta_u = .25$, $p < .05$; $\beta_v = .18$, $p < .05$). These results support journalism scholars' conceptual assertions that cuts in newsroom investments adversely affect newspaper performance (Overholser 2004).

In summary, the empirical analyses show that market data support the proposed model, furnish strong evidence of the presence of CMEs, and shed light on the indirect marketing elasticities induced by CMEs. The empirical results not only are new to marketing literature but also are of considerable value to the newspaper firm in particular and the daily newspaper industry in general.

Replication. To further validate the model, we obtained additional data from a different privately held media company. The data set contains information on revenues from attractors (readers) and suitors (advertisers), as well as investments in the newsroom and sales force. Applying the KF approach, we repeated the analysis in the previous subsections. Again we found that the proposed platform firm response model fits the data well, outperforms competing specifications (i.e., those without CMEs and/or with uncorrelated error terms), and forecasts satisfactorily. We summarize the main findings briefly.

First, we find evidence of reinforcing CMEs ($\theta_{AS} = .27$, $p < .05$; $\theta_{SA} = .18$, $p < .05$). Second, we again find moderate carryover values in the attractor and suitor revenues ($\lambda_A = .48$, $p < .05$; $\lambda_S = .50$, $p < .05$). Third, the newsroom investments significantly affect attractor revenues ($\beta_u = .67$, $p < .05$), and sales force investments significantly affect suitor revenues ($\beta_v = .34$, $p < .05$). Because of the implicit interaction effects in the multiplicative model, newsroom investments have an indirect impact on suitor revenues, and sales force investments have an indirect impact on attractor revenues. Fourth, we find evidence of a declining trend in attractor revenues ($\gamma_1 = -.003$, $p < .01$), an increase in the same in the November–December months ($\gamma_2 = .09$, $p < .01$), and a fall in suitor revenues in July ($\gamma_3 = -.06$, $p < .01$). This replication of our previous findings—using a different newspaper of a different company—enhances our confidence in the validity of our proposed model. Next, we show how this validated model can determine dynamically optimal marketing budgeting and allocations.

A MANAGERIAL DECISION TOOL AND REAL-WORLD APPLICATION

Problem Motivation

Two-sided media firms such as daily newspapers, magazines, and radio stations must decide how much they should invest in news quality, directed at readers or listeners, and in sales force effort, directed at advertisers, over some finite planning horizon. However, many newspapers tend to view investments in the newsroom as costs that they can cut to improve profits. Such cutbacks are questionable, because they are not based on a systematic assessment of the long-term consequences for circulation (attractor revenues) and, in turn, advertising revenue (suitors). In contrast, the model-based decision tool we develop accounts for both long-term effects (λs) and CMEs (θs) to derive optimal marketing investment trajectories over a prespecified planning horizon.

Decision Tool

The platform firm's goal is to maximize the total profit $J(u, v)$ over the planning horizon T. More formally, we capture this goal with the objective function:

(10) $\qquad$ Maximize $\quad J(u, v) = \sum_{t=1}^{T} e^{-\rho t} \pi(A_t, S_t, u_t, v_t)$,

where $\pi(A, S, u, v) = m_A A + m_S S - u - v$. To maximize Equation 10, we apply Hamilton's maximum principle (e.g., Kamien and Schwarz 1992; Sethi and Thompson 2006) and derive the necessary conditions (see Appendix B), the optimal controls,

(11) $\qquad \begin{bmatrix} w_{ut}^* \\ w_{vt}^* \end{bmatrix} = \begin{bmatrix} Ln(\beta_u \mu_{1t}) \\ Ln(\beta_v \mu_{2t}) \end{bmatrix}$,

and the co-state dynamics,

(12) $\begin{bmatrix} \mu_{1t} - \mu_{1,t-1} \\ \mu_{2t} - \mu_{2,t-1} \end{bmatrix} = \begin{bmatrix} \rho + 1 - \lambda_A & -\theta_{AS} \\ -\theta_{SA} & \rho + 1 - \lambda_S \end{bmatrix} \begin{bmatrix} \mu_{1,t-1} \\ \mu_{2,t-1} \end{bmatrix}$
$\qquad\qquad + \begin{bmatrix} -m_A e^{Y_{At}} \\ -m_S e^{Y_{St}} \end{bmatrix}$,

where (w_{ut}^*, w_{vt}^*) represent the optimal investments toward attractors and suitors, and (μ_{1t}, μ_{2t}) are the co-state variables corresponding to the attractor and suitor sales dynamics, respectively. To find the best investments strategies, we must not only solve jointly the optimal controls in Equation 11, the co-state equations in Equation 12, and the state equations in Equation 5 but also account for the initial conditions given by

(13) $\qquad \begin{bmatrix} Y_{A0} \\ Y_{S0} \end{bmatrix} = \begin{bmatrix} \ln(\bar{A}) \\ \ln(\bar{S}) \end{bmatrix}$

and the terminal conditions given by

(14) $\begin{bmatrix} \mu_1^* \\ \mu_2^* \end{bmatrix} = \dfrac{1}{(\rho + 1 - \lambda_A)(\rho + 1 - \lambda_S) - \theta_{AS}\theta_{SA}}$
$\qquad\qquad \begin{bmatrix} \rho + 1 - \lambda_A & -\theta_{AS} \\ -\theta_{SA} & \rho + 1 - \lambda_S \end{bmatrix} \begin{bmatrix} m_A \bar{A} \\ m_S \bar{S} \end{bmatrix}$,

which represent the steady state of Equation 12, evaluated at the market conditions. Considering these initial and terminal conditions, the resulting dynamic maximization problem is a "two-point boundary value" problem.

To solve this problem, we adapt the numerical algorithm proposed by Naik, Raman, and Winer (2005). First, we augment the state vector to contain both the state and co-state variables. That is, we define the 4×1 vector $z = (Y_A, Y_S, \mu_1, \mu_2)'$, whose transition is given by

$$(15) \quad \begin{bmatrix} Y_{At} - Y_{A,t-1} \\ Y_{St} - Y_{S,t-1} \\ \mu_{1t} - \mu_{1,t-1} \\ \mu_{2t} - \mu_{2,t-1} \end{bmatrix}$$

$$= \begin{bmatrix} -(1-\lambda_A) & \theta_{SA} & 0 & 0 \\ \theta_{AS} & -(1-\lambda_S) & 0 & 0 \\ 0 & 0 & \rho+1-\lambda_A & -\theta_{AS} \\ 0 & 0 & -\theta_{SA} & \rho+1-\lambda_S \end{bmatrix}$$

$$\begin{bmatrix} Y_{A,t-1} \\ Y_{S,t-1} \\ \mu_{1,t-1} \\ \mu_{2,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut}^* + \gamma_1 t \\ \beta_v w_{vt}^* + \gamma_2 D_{1,t} + \gamma_3 D_{2,t} \\ -m_A e^{Y_{A,t-1}} \\ -m_S e^{Y_{S,t-1}} \end{bmatrix}.$$

Note that Equation 15 is nonlinear, due to the exponential terms and the optimal controls (w_{ut}^*, w_{vt}^*). Then, using Equation 15, we define a term E_t as

$$(16) \quad E_t = z_t - z_{t-1} - g(z_{t-1}, \hat{\Psi}),$$

where $\hat{\Psi}$ contains the estimated parameters, and the nonlinear function $g(\cdot)$ is informed by Equation 15. Note that Equation 16 generates $(T-1)$ equations, each of which is a 4×1 vector.

Second, we incorporate the initial and terminal conditions. Using Equation 13, we obtain two equations with $E_1 = k_1'[z_1 - z_0 - g(z_0, \hat{\Psi})]$, where $k_1 = (1, 1, 0, 0)'$. Similarly, with Equation 14, we obtain two more equations from $E_{T+1} = k_2'[z_{T+1} - z_T - g(z_T, \hat{\Psi})]$, where $k_2 = (0, 0, 1, 1)'$. Note that we have equations $E_1, E_2, \ldots, E_T, E_{T+1}$, where E_1 and E_{T+1} are 2×1 vectors, and the other E_t are 4×1 vectors. By stacking them, one below another, we create a long vector G of dimension $(4T) \times 1$. This resulting vector G is a function of $(4T) \times 1$ variables $x = \text{vec}(z_1, z_2, \ldots, z_T)$.

Third, to obtain the optimal state and co-state trajectories, we solve the large system of nonlinear difference equations $G(x) = 0$ by applying a quasi-Newton root-finding procedure (e.g., eqSolve in Gauss 7.0). We initiate this procedure by starting from the actual sales trends and the co-state trajectories implied by actual sales, margin, and spending data. Fourth, using the converged solution, we use Equation 11 to compute the optimal marketing investments over time. We thus can demonstrate the application of this decision tool.

Real-world application. The newspaper company provided data about not only sales and marketing investments but also margins from sales to subscribers and to advertisers. We use the first 90 months of the data for model calibration and the last 30 months as the implementation period. Applying the decision tool to these market data, we computed the optimal trajectories of u_t^* and v_t^* for the 30-month period, the corresponding attractor and suitor sales trajectories A_t^* and S_t^*, and the resulting optimal total profits trajectory π_t^*. We compared the optimal investments with the actual spends over time (u_t and v_t), the associated revenues A_t and S_t, and the profits π_t. Our results indicated that the managers, on average, were underspending in the previous 30-month period on the sales force ($v^*/v = 1.25$) and much more so in the newsroom ($u^*/u = 1.43$). By following optimal spending paths instead, the firm could significantly increase revenues and profit: Attractor revenue could increase by 50%, suitor revenue by 51%, and overall profit by 28%.

In summary, the newspaper could increase investments in the newsroom (i.e., investments that increase readership, which may not necessarily increase objective news "quality") and achieve higher profitability, a finding that resonates with many journalism scholars (e.g., Lacy and Martin 2004; Rosentiel and Mitchell 2004). The newspaper's senior management reviewed these recommendations from the decision tool, were convinced of its value, and decided to increase investments in the newsroom by $500,000, representing an average monthly increase of 18%. This decision not only represented a significant reversal in direction for the firm but also was contrary to the current national trends of slashing newsroom investments. Because the newspaper's managers could only obtain additional funds of $500,000, they decided to invest all of it in the newsroom, because they felt they were closer to the optimal sales force investment than they were to the optimal newsroom investment. Management is currently sourcing additional funding to invest in the sales force department as well.

Finer or coarser temporal aggregation. Managers' decision calendars may follow a different frequency than the one used to estimate the model. For example, managers may make marketing budgeting decisions over a coarser (e.g., quarterly, yearly) or finer (e.g., weekly) frequency. We augment our marketing budgeting algorithm to resolve situations when the model calibration and decision frequencies differ. Specifically, Equation 16 suggests that the change of z_t from time point $t-1$ to t is given by $g(z_{t-1}, \hat{\Psi})$. This version assumes that the difference between $t-1$ and t is one month. To allow for marketing investment decisions on a finer or coarser decision interval, we can choose R grid points in the planning calendar, denoting them $r = 1, 2, \ldots, R$, and rewriting Equation 16 as

$$(17) \quad E_r = z_r - z_{r-1} - hg(z_{r-1}, \hat{\Psi}),$$

where $h = t_r - t_{r-1}$ denotes the time interval between the two grid points $r-1$ and r. The use of Equation 17 transposes the problem into one in which the optimal solutions are found on r points in the planning calendar. Depending on the choice of r (e.g., weekly, quarterly), we can obtain solutions over a finer ($r <$ one month) or coarser ($r > 1$ one month) planning calendar.

ANALYTICAL INSIGHTS

To gain normative insights to guide the marketing investments of an ongoing platform firm with two end-user

groups, we adopt the long-run perspective of a firm that expects to remain in business for the foreseeable future. We analyze optimal investments under steady-state conditions. Specifically, we maximize discounted profits over a long planning horizon ($T \to \infty$). Appendix C shows that the optimal marketing investments are then expressed as

$$(18) \quad \begin{bmatrix} u_t^* \\ v_t^* \end{bmatrix} = \frac{1}{(\rho+1-\lambda_A)(\rho+1-\lambda_S)-\theta_{AS}\theta_{SA}}$$
$$\begin{bmatrix} [\beta_u(m)]_A A_t^*(\rho+1-\lambda_S) + m_S S_t^* \theta_{AS} \\ [\beta_v(m_A A_t^* \theta_{SA} + m_S)] S_t^*(\rho+1-\lambda_A)) \end{bmatrix}.$$

Equation 18 reveals that the platform firm's optimal marketing investments differ from those for a classic firm. For the platform firm, the marginal return of each marketing investment (u and v) depends on both cross-market effects and dynamic effects on both sides of the market. Consequently, the optimal marketing investment-to-sales ratio depends on interdependencies in the system.

A comparative statics analysis of Equation 18 shows how the presence of CMEs alters the investment levels compared with a classic firm with the same sales carryover dynamics and discount rate but CMEs close to zero (θ_{SA}, $\theta_{AS} \to 0$).

R$_1$: All else being equal, optimal marketing efforts by reinforcing platform firms (both θ_{AS} and $\theta_{SA} > 0$) directed at both attractors and suitors are greater than those by classic firms (θ_{AS}, $\theta_{SA} \to 0$).

An example of a reinforcing platform is a local newspaper with ad-loving readers (e.g., Sonnac 2000). This first result offers the insight that when CMEs are mutually reinforcing, a profit-maximizing platform firm spends more on marketing, not less, than its counterpart classic firm, because managers should account for the long-term benefits of own market carryovers and CMEs.

R$_2$: All else being equal, optimal marketing efforts by counteractive platforms ($\theta_{AS} > 0$, $\theta_{SA} < 0$) directed at attractors are greater than those by classic firms (θ_{AS}, $\theta_{SA} \to 0$), provided the margin ratio m_S/m_A exceeds the critical value m*.

This result reveals that an important trade-off exists in marketing by counteractive platforms. Increasing marketing toward attractors (u) leads to an increase in attractor revenue (A) and, subsequently, an increase in suitor revenue (S) through the attraction effect (θ_{AS}). However, an increase in suitors, and therefore in suitor revenues, deters the long-term revenue from attractors, such as in a setting with ad-avoiding newspaper and magazine readers (Sonnac 2000). The amount of loss depends on the magnitude of the negative suitor effect θ_{SA} and the long-term own effect of attractors (λ_A).

The critical margin ratio is given by $[((A_0^* - A_C^*)(\rho+1-\lambda_A)(\rho+1-\lambda_S)-A_0^*\theta_{AS}\theta_{SA}/S_c^*(\rho+1-\lambda_A)\theta_{AS})$, where (S_c^*, A_c^*, A_o^*) are counteractive suitor, counteractive attractor sales, and classic attractor sales, respectively. At this ratio, the long-term profit contribution of the suitors' revenue outweighs the lost contribution due to the lower attractor revenues that the counteractive effect induces. The critical margin ratio increases as the suitor effect or carryover effect increases, and it decreases with the discount rate.

Result 2 thus suggests that rather than indiscriminately adding attractors, managers of counteractive platforms should tailor their marketing messages to gain attractors who may be more tolerant to suitors. For example, prior research indicates that significant heterogeneity in ad avoidance exists among the potential readers of magazines and newspapers (Sonnac 2000). In such situations, managers should target market segments that are less ad averse.

R$_3$: All else being equal, optimal marketing effort by a counteractive platform ($\theta_{AS} > 0$, $\theta_{SA} < 0$) directed at suitors is lower than that of a classic firm (θ_{AS}, $\theta_{SA} \to 0$).

That is, although the effect of v in increasing the number of suitors may be large, the negative value of θ_{SA} reduces its overall long-term effectiveness, which reduces its optimal spending level. Result 3 has implications for investments in the ad selling effort of platforms. For example, news radio stations commonly employ salespeople to sell "piggyback" slots to retailers, with multiple slots scheduled back-to-back. Although they significantly increase revenue for the station, they increase the number of ads heard during a program and increase the clutter of messages (Warner and Buchman 1991). Increased clutter contributes to wasted coverage (i.e., listeners not buying from advertisers) and/or high turnover (i.e., listeners switching stations). In such situations, increasing investments in the sales force may not be optimal for the station, even if salespeople are effective in selling piggyback slots to retailers.

CONCLUSION

Marketing managers bear the responsibility for planning their investment budget and its allocation optimally and for demonstrating that these investments generate appropriate returns for the firm. Although considerable research on this topic exists, literature so far has largely ignored resource allocation by platform firms operating in two-sided markets characterized by cross-market effects (CMEs). This gap in research motivated us to investigate two-sided platform firms' marketing decisions both theoretically and empirically.

We developed a platform-firm response model that takes into account the quintessential features of the two-sided market, including demand interdependence, such that zero demand from attractors results in zero demand from suitors (but not vice versa). Subsequently, we estimated and validated the proposed model using data from an archetypal platform firm, namely, a daily newspaper company with two end-user groups: readers and advertisers. We also replicated the proposed model using data from another newspaper platform firm. Both the original and replication analyses revealed the presence of dynamic CMEs between these newspapers' readers and advertisers. We then developed a novel algorithm to assist platform firm managers determine dynamically optimal marketing investment paths over a finite planning horizon. Finally, we conducted a normative analysis and derived new propositions about how optimal marketing toward end-user groups is affected by

CMEs that arise in platform firm. The key takeaways for managers and academics are as follows:

Takeaway 1: In developing a platform firm response model, it is crucial to take into account dynamic demand interdependence between the firm's two markets, as well as the idea that zero demand from attractors results in zero demand from suitors (but not vice versa). We develop and empirically validate such a platform firm market response model in the context of daily newspapers.

Takeaway 2: Our marketing mix algorithm solves the implied nonlinear boundary value problem to obtain dynamically optimal marketing budgeting for managers. Our results show that the presence of CMEs substantially increases the net long-term worth of the newspaper's spending on newsroom quality, because this investment attracts readers and in turn achieves higher advertiser revenues. Thus, newspapers should increase investments in news quality, which is contrary to the practice of cutting newsroom investments to shore up profits, as followed by many troubled newspaper companies today.

Takeaway 3: Our recommendation based on the proposed marketing-mix algorithm was formally accepted by the newspaper management, which increased the annual newsroom budget by $500,000 (a substantial increase of 18%).

Takeaway 4: In general, it is crucial for platform managers to account for both CME and carryover effects when making marketing investment decisions. The CME structure may imply higher marketing investments in the case of reinforcing platforms (R_1); in counteractive platforms, managers should weigh the gain from adding suitors against the loss of some attractors when setting marketing investment levels (R_2).

Takeaway 5: The interplay between own-market and cross-market effects may make it optimal for a platform firm to invest in marketing to a lower-margin group (R_2).

We hope that platform managers, especially those from troubled newspaper companies, use our proposed model-based approach to determine dynamically optimal marketing investments toward the two sides of their firms' markets.

We conclude by identifying four avenues for research. First, our model applies to monopoly platform marketing, and therefore, the results do not necessarily generalize to competitive markets. Additional research should extend our analyses to competitive markets. (We thank an anonymous reviewer for this suggestion.) Second, another extension would incorporate time-varying CMEs, allowing for both reinforcing and counteractive effects over different ranges of the data. Third, researchers should apply the model to settings with possibly increasing returns to scale. Fourth, estimations and applications of our model in other platform settings would be worthwhile. For example, one research objective could be to investigate optimal marketing investment decisions over time in television broadcast markets, where existing evidence indicates viewers are ad averse and take deliberate actions to avoid ads (e.g., Gustafson and Siddharth 2007).

APPENDIX A: TEST FOR EXOGENEITY

Applying Engle, Hendry, and Richard's (1983) approach, we test for the exogeneity of newsroom and sales force investments. Let $p_1(Y_A, w_u)$ be the joint density of attractor revenues and newsroom investments, $p_2(Y_A \mid w_u)$ denote the conditional density of attractor revenues given newsroom investments, and $p_3(w_u)$ represent the marginal density. We factorize $p_1(Y_A, w_u) = p_2(Y_A \mid w_u) \times p_3(w_u)$, and

weak exogeneity means that a precise specification of $p_3(\cdot)$ is not needed and no loss of information occurs when the estimation is based on the conditional density $p_2(\cdot)$. To verify this claim, we first estimated marginal models of newsroom and sales force investments (see Engle, Hendry, and Richard 1983; Naik, Raman, and Winer 2005):

(A1) $\qquad w_{ut} = \alpha_{0n} + \alpha_{in}w_{ut-ni} + \alpha_{jn}Y_{At-nj} + \alpha_{kn}Y_{St-nk} + \varepsilon_{u,t}.$

(A2) $\qquad w_{vt} = \alpha_{0s} + \alpha_{is}w_{vt-si} + \alpha_{js}Y_{St-sj} + \alpha_{ks}Y_{At-sk} + \varepsilon_{v,t}.$

We determined ni, nj, nk, si, sj, and sk on the basis of the AIC. We computed the residuals pertaining to newsroom and sales force investments (i.e., $\varepsilon_{w_{ut}}$ and $\varepsilon_{w_{vt}}$). Next, we obtained the residuals from the estimated system of sales revenues from the KF estimation. We then examined the correlations between the marginal model and the conditional model residuals. For newsroom investments, we examined the correlations from the marginal model of newsroom investments and the subscriptions revenue equation. The resulting correlation (*p*-value) is −.059 (.532). Because it lacks significance at the 95% confidence level, newsroom investments are weakly exogenous. Similarly, for sales force investments, we examined the correlations from the marginal model of sales force investments and the ad revenue equation. The resulting correlation (*p*-value) is .067 (.476). It lacks significance at the 95% confidence level, so sales force investments also are weakly exogenous. Thus, we find that both newsroom and sales force investments are weakly exogenous. The empirical results are valid in the sense of efficient estimation (Engle, Hendry, and Richard 1983).

APPENDIX B: DYNAMIC OPTIMIZATION

Given the presence of CMEs (θ_{AS} and θ_{SA}), how should platform firm managers determine their overall marketing budget trajectory and the budget allocation across attractors and suitors? Let u_t and v_t denote the marketing investments toward attractors and suitors, respectively. For a given discount rate ρ, the platform firm seeks to find the investment levels that maximize the total discounted profits, expressed as

(B1) $\qquad$ Maximize $\quad J(u, v) = \sum_{t=0}^{T-1} e^{-\rho t} \pi(A_t, S_t, u_t, v_t),$

(B2) $\qquad$ where $\quad \pi(A_t, S_t, u_t, v_t) = m_A A_t + m_S S_t - u_t - v_t,$

subject to the dynamic market response functions of the platform firm:

(B3) $\qquad\qquad A_t = A_{t-1}^{\lambda_A} \tilde{S}_{t-1}^{\theta_{SA}} u_t^{\beta_u} e^{\kappa_1 \varphi_t}, \text{ and}$

(B4) $\qquad\qquad S_t = \tilde{S}_{t-1}^{\lambda_S} A_{t-1}^{\theta_{AS}} v_t^{\beta_v} e^{\kappa_2 \phi_t},$

where $\tilde{S}_{t-1} = \text{Max}\{S_{t-1}, \varphi_t\}$ and $\varphi_t = \{1 \text{ when } S_{t-1} = 0, \text{ and } 0 \text{ when } S_{t-1} > 0\}$.

Because Equations B3 and B4 are nonlinear, we transform them using logarithms:

(B5) $\qquad \begin{bmatrix} Y_{At} \\ Y_{St} \end{bmatrix} = \begin{bmatrix} \lambda_A & \theta_{SA} \\ \theta_{AS} & \lambda_S \end{bmatrix} \begin{bmatrix} Y_{A,t-1} \\ Y_{\tilde{S},t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut} + \kappa_1 \phi_t \\ \beta_v w_{vt} + \kappa_2 \phi_t \end{bmatrix},$

where Y_{At} and Y_{St} represent log-transformed attractor and suitor sales, respectively, and $w_{u,t}$ and $w_{v,t}$ represent log-transformed investments toward the attractors and suitors,

respectively. By including trend and seasonality terms, we obtain

(B6) $\begin{bmatrix} Y_{At} \\ Y_{St} \end{bmatrix} = \begin{bmatrix} \lambda_A & \theta_{SA} \\ \theta_{AS} & \lambda_S \end{bmatrix} \begin{bmatrix} Y_{A,t-1} \\ Y_{\tilde{S},t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut} \\ \beta_v w_{vt} \end{bmatrix}$
$+ \begin{bmatrix} \gamma_1 t + \kappa_1 \phi_t \\ \gamma_2 D_{1,t} + \gamma_3 D_{2,t} + \kappa_2 \phi_t \end{bmatrix}.$

Next, subtracting $Y_{A,t-1}$ from the first row and $Y_{\tilde{S},t-1}$ from the second row of Equation B6, we get

(B7) $\begin{bmatrix} Y_{At} - Y_{A,t-1} \\ Y_{St} - Y_{\tilde{S},t-1} \end{bmatrix} = \begin{bmatrix} \lambda_A - 1 & \theta_{SA} \\ \theta_{AS} & \lambda_S - 1 \end{bmatrix} \begin{bmatrix} Y_{A,t-1} \\ Y_{\tilde{S},t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut} \\ \beta_v w_{vt} \end{bmatrix}$
$+ \begin{bmatrix} \gamma_1 t + \kappa_1 \phi_t \\ \gamma_2 D_{1,t} + \gamma_3 D_{2,t} + \kappa_2 \phi_t \end{bmatrix},$

which can be expressed as follows when $S_{t-1} > 0$:

(B8) $\begin{bmatrix} \Delta Y_{At} \\ \Delta Y_{St} \end{bmatrix} = \begin{bmatrix} -(1-\lambda_A) & \theta_{SA} \\ \theta_{AS} & -(1-\lambda_S) \end{bmatrix} \begin{bmatrix} Y_{A,t-1} \\ Y_{S,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut} \\ \beta_v w_{vt} \end{bmatrix}$
$+ \begin{bmatrix} \gamma_1 t \\ \gamma_2 D_{1,t} + \gamma_3 D_{2,t} \end{bmatrix}.$

If $S_{t-1} = 0$ for some t, then we replace θ_{SA} by 0 and λ_S by unity and add $\kappa_1 \phi_t$ and $\kappa_2 \phi_t$ to Equation B8.

To solve this dynamic optimization problem, we apply the discrete-time maximum principle to derive the optimal effort levels. When $S_{t-1} > 0$, the Hamiltonian at each instant t is

(B9) $H_t = [m_A e^{Y_{At}} + m_S e^{Y_{St}} - e^{w_{u,t}} - e^{w_{v,t}}]$
$+ \mu_{1,t+1}[-(1-\lambda_A)Y_{At} + \theta_{SA} Y_{St} + \beta_u w_{ut} + \gamma_1 t]$
$+ \mu_{2,t+1}[-(1-\lambda_S)Y_{St} + \theta_{AS} Y_{At} + \beta_v w_{vt}$
$+ \gamma_2 D_{1,t} + \gamma_3 D_{2,t}],$

where $\mu_{1,t+1}$ and $\mu_{2,t+1}$ are the co-state variables corresponding to the two state equations. If $S_{t-1} = 0$ for some t, then we replace θ_{SA} by 0 and λ_S by unity and add $\kappa_1 \phi_t$ and $\kappa_2 \phi_t$ to Equation B9. The conditions for optimality are

(B10) $\dfrac{\partial H_t}{\partial w_{u,t}} = 0 \quad \text{and} \quad \dfrac{\partial H_t}{\partial w_{v,t}} = 0,$

(B11) $\Delta \mu_{1t} = \mu_{1t+1} - \mu_{1t} = \rho \mu_{1t} - \dfrac{\partial H_t}{\partial Y_{At}}$
$= \rho \mu_{1t} - m_A e^{Y_{At}} + \mu_{1t}(1-\lambda_A) - \mu_{2t} \theta_{AS},$ and

(B12) $\Delta \mu_{2t} = \mu_{2,t+1} - \mu_{2t} = \rho \mu_{2t} - \dfrac{\partial H_t}{\partial Y_{St}}$
$= \rho \mu_{2t} - m_S e^{Y_{St}} + \mu_{2t}(1-\lambda_S) - \mu_{1t} \theta_{SA}.$

From Equation B10, we obtain the optimal controls:

(B13) $w_{ut}^* = Ln(\mu_{1t}\beta_u), \quad \text{and} \quad w_{vt}^* = Ln(\mu_{2t}\beta_v),$

which can be exponentiated to transform back to the actual marketing investments. To derive analytical insights, we obtain the stationary co-state variables by setting Equations B11 and B12 to zero,

(B14) $\begin{bmatrix} \mu_{1t}^* \\ \mu_{2t}^* \end{bmatrix} = \dfrac{1}{(\rho+1-\lambda_A)(\rho+1-\lambda_S) - \theta_{AS}\theta_{SA}}$
$\begin{bmatrix} \rho+1-\lambda_S & \theta_{AS} \\ \theta_{SA} & \rho+1-\lambda_A \end{bmatrix} \begin{bmatrix} m_A A_t^* \\ m_S S_t^* \end{bmatrix},$

where A_t^* and S_t^* represent the optimal attractor and suitor values, respectively. Substituting Equation B14 into Equation B13, we obtain the optimal marketing investments:

(B15) $\begin{bmatrix} u_t^* \\ v_t^* \end{bmatrix} = \dfrac{1}{(\rho+1-\lambda_A)(\rho+1-\lambda_S) - \theta_{AS}\theta_{SA}}$
$\begin{bmatrix} \beta_u(m_A A_t^*(\rho+1-\lambda_S) + m_S S_t^* \theta_{AS}) \\ \beta_v(m_A A_t^* \theta_{SA} + m_S S_t^*(\rho+1-\lambda_A)) \end{bmatrix}$

APPENDIX C: PROOFS OF ANALYTICAL RESULTS

Proof R1

Using Equation 18 in the article,

(C1) $u_r^* = \dfrac{\beta_u(m_A A_r^*(\rho+1-\lambda_S) + m_S S_r^* \theta_{AS})}{(\rho+1-\lambda_A)(\rho+1-\lambda_S) - \theta_{AS}\theta_{SA}},$
$\theta_{AS}, \ \theta_{SA} > 0, \ \text{and}$

(C2) $u_0^* = \dfrac{\beta_u(m_A A_0^*)}{(\rho+1-\lambda_A)}, \quad \theta_{AS}, \ \theta_{SA} \to 0,$

where the subscript r refers to reinforcing, and the subscript 0 refers to classic firm. Rewriting Equation C2, we get

(C3) $u_0^* = \dfrac{\beta_u(m_A A_0^*)(\rho+1-\lambda_S)}{(\rho+1-\lambda_A)(\rho+1-\lambda_S)}.$

A comparison of Equations C3 and C1 reveals that the denominator of Equation C3 is higher than that of Equation C1 ($\theta_{AS} > 0, \theta_{SA} > 0$), whereas the numerator of Equation C3 is lower than that of Equation C1. Furthermore, $A_r^* > A_0^*$, and $u_r^* > u_0^*$, thus proving the claim.

Proof R2

Using Equation 18 in the article,

(C4) $u_c^* = \dfrac{\beta_u(m_A A_c^*(\rho+1-\lambda_S) + m_S S_c^* \theta_{AS})}{(\rho+1-\lambda_A)(\rho+1-\lambda_S) - \theta_{AS}\theta_{SA}},$
$\theta_{AS} > 0, \ \theta_{SA} < 0, \ \text{and}$

(C5) $u_0^* = \dfrac{\beta_u(m_A A_0^*)}{(\rho+1-\lambda_A)}, \quad \theta_{AS}, \ \theta_{SA} \to 0,$

where the subscript c refers to counteractive and the subscript 0 refers to classic firm. A comparison of Equations C4 and C5 reveals that $u_c^* > u_0^*$ only when

(C6) $m_S S_c^*(\rho+1-\lambda_A)\theta_{AS}$
$> m_A((A_0^* - A_C^*)(\rho+1-\lambda_A)(\rho+1-\lambda_S) - A_0^*\theta_{AS}\theta_{SA}),$

which is equivalent to

(C7) $\dfrac{m_S}{m_A} > \dfrac{(A_0^* - A_C^*)(\rho+1-\lambda_A)(\rho+1-\lambda_S) - A_0^*\theta_{AS}\theta_{SA}}{S_c^*(\rho+1-\lambda_A)\theta_{AS}} = m^*.$

This inequality shows that $u_c^* > u_0^*$ when $m_S/m_A > m^*$, as posited.

Proof R3

Using Equation 18 in the article,

(C8) $v_c^* = \dfrac{\beta_v(m_A A_c^* \theta_{SA} + m_S S_c^*(\rho+1-\lambda_A))}{(\rho+1-\lambda_A)(\rho+1-\lambda_S) - \theta_{AS}\theta_{SA}},$
$\theta_{AS} > 0, \ \theta_{SA} < 0, \ \text{and}$

(C9) $v_0^* = \dfrac{\beta_v(m_S S_0^*)}{(\rho+1-\lambda_S)}, \quad \theta_{AS}, \ \theta_{SA} \to 0.$

Rewriting Equation C9, we get

$$\text{(C10)} \qquad v_0^* = \frac{\beta_v (m_S S_0^*)(\rho + 1 - \lambda_A)}{(\rho + 1 - \lambda_A)(\rho + 1 - \lambda_S)}.$$

A comparison of Equations C10 and C8 reveals that the denominator of Equation C10 is lower than that of Equation C8 ($\theta_{AS} > 0$, $\theta_{SA} < 0$), whereas the numerator of Equation C10 is higher than that of Equation C8. Therefore, $v_c^* < v_0^*$, which proves the claim.

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