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Long-term Profit Impact Of Integrated Marketing Communications Program

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Abstract

The concept of Integrated Marketing Communications (IMC) emphasizes the role of synergy, which arises when the combined effect of multiple activities exceeds the sum of their individual effects. In this paper, we investigate the effects of synergy on the profitability of IMC programs in uncertain markets. We develop a dynamic multimedia model that incorporates both synergy and uncertainty, and use it to determine the optimal IMC program. Our results generalize previous findings to uncertain markets, illuminate the profit implications of IMC programs, and explain the catalytic effects of synergy in IMC contexts. Specifically, we find that the expected long-term profit of the advertised brand increases as synergy increases. Furthermore, managers should allocate a non-zero budget to a catalytic activity even if it is completely ineffective. Finally, these findings continue to hold in an uncertain duopoly market.

KEYWORDS: Marketing Communications, Stochastic Optimal Control, Synergy

1. INTRODUCTION

The proliferation of media into specialized magazines, cable programming, in-transit advertisements and the Internet has fragmented the readership or viewing audience, placing greater demands on consumers' attention, and thereby eroding the influence of mass advertising. In response to media proliferation, marketers are attempting to increase the impact of their communications program through an Integrated Marketing Communications (IMC) perspective, hoping to harness synergies between media (see, e.g., Belch and Belch 1998, p. 11). Synergy arises when the combined effect of two activities exceeds the sum of their individual effects, a phenomenon in which "the whole is greater than the sum of the parts." Naik and Raman (2003) show how to estimate synergy and understand its effects on media budget and allocation. Their analysis reveals that advertisers should increase the media budget in the presence of synergy and allocate the increased media budget disproportionately in favor of the less effective medium. More importantly, using Naik and Raman's (2003) IMC model, Schultz and Pilotta (2004) further enhance our understanding of "how media advertising works in the interactive, networked, global systems found in the 21st century media marketplace."

This IMC model, however, assumes that the impact of media spending on sales is deterministic, but this assumption is sometimes untenable, for instance in turbulent, volatile markets in which uncontrollable factors affect sales. How then should advertisers plan IMC programs under market uncertainty? What are the long-term profit implications of the IMC program in uncertain markets? The need for research on the implications of uncertainty for long-term profitability has been stressed by the MAX initiative — Managing Advertising Expenditures for Financial Performance — sponsored by both the American Association of Advertising Agencies and the Marketing Science Institute (Farris, Shames and Reibstein 1998). We respond to this need by elucidating the impact of synergy on profitability of IMC programs in uncertain markets. In addition, we augment the extant literature with insights into the catalytic effects of synergy.

To this end, we include uncertainty in the Naik and Raman (2003) model of IMC via the Wiener process in order to represent error in their continuous-time dynamic model. Applying stochastic optimal control theory, we then derive the optimal IMC program incorporating cross-media synergy under uncertainty. Next, on the basis of comparative static analyses, we find that the expected long-term profit of the advertised brand increases as cross-media synergy increases. Furthermore, cross-media synergy neither increases nor decreases the variability in long-term profit. Hence, ad agencies and brand managers can enhance synergies between multiple media to increase the brand's long-term profitability, without influencing the variability of profits.

Our fundamental result pertains to the importance of ancillary activities in the communications mix, even when such ancillary activities may appear to have negligible impact on sales. Specifically, advertisers should use activities such as event sponsorship, free-samples and collaterals, in-transit advertising or merchandising because these ancillary activities enhance the effectiveness of primary activities through synergistic interactions. For example, BMW places its sports car in Bond movies not because product placement (an ancillary activity) directly increases car sales, but because it interacts with other marketing activities, thereby enhancing visibility of the brand. In other words, ancillary activities are catalysts that positively influence sales growth although they lack *direct* sales impact. We later define these so-called catalytic effects of synergy, and explain why managers should allocate a non-zero budget to an ancillary activity even if it is ineffective, as measured by its *direct* linkage to sales.

The rest of the paper is organized as follows. Section 2 reviews the extant literature on integrated marketing communications. Section 3 incorporates uncertainty in the IMC model and formulates the budgeting problem. Section 4 generalizes the previous results and presents new results on long-term profitability and catalytic effects of synergy. Section 5 further extends the IMC model and generalizes the findings to duopoly markets; it also discusses the effects of uncertainty and advertising.

2. LITERATURE REVIEW

An abundant literature on the effects of advertising on sales exists in marketing science. However, knowledge about the *joint* effects of multimedia advertising is quite limited (see Gatignon 1993, Mantrala 2002). For example, in their comprehensive literature review, Feichtinger, Hartl and Sethi (1994, p. 219) observe that, "with a few exceptions, the models assume … single advertising medium. This was already noted by Sethi (1977), and this critical remark is still valid for the literature published subsequently."

To gain insights into joint effects of multimedia advertising, a consortium of radio network companies conducted a field study by sampling 500 adults, ages 20-44, and across 10 locations in Britain. The main findings indicate that 73% of the participants remembered prime visual elements of TV ads upon hearing radio commercials. In addition, 57% re-lived the TV ads while listening to the radio advertisement. Thus, radio ads reinforce the imagery created by TV commercials, resulting in synergy between television and radio advertising.[1] In controlled

[1] For additional information, contact the Radio Advertising Bureau or visit www.rab.co.uk.

laboratory experiments, Edell and Keller (1999) analyze interactions between TV and print advertising, and find significant effects of cross-media synergy. More recently, Naik and Raman (2003) investigate the effects of synergy in an empirically validated model of multimedia advertising, and provide several insights based on their theoretical analyses.

Specifically, Naik and Raman (2003) estimate a dynamic sales response model using market data on Dockers® brand advertising, and establish the presence of synergy between TV and print ads in consumer markets. Then they theoretically show that media budget increases as the magnitude of synergy increases. This finding provides a new perspective on the frequently debated issue of over-advertising. The marketing literature (see Hanssens et al. 1998, p. 260) suggests that advertisers tend to overspend on advertising. However, a response model that ignores the effects of synergy understates the optimal budget. Hence, what *appears* to be overspending would represent an appropriate spending level when we account for synergy between multiple media.

More importantly, Naik and Raman (2003) find that budget allocation across media differs qualitatively in the presence of synergy. As synergy between two media increases, the proportion of budget allocated to the *more* (less) effective medium *decreases* (increases). To understand this counter-intuitive result, consider a market with no synergy. Here each medium independently increases brand sales, and so the optimal spending on each medium depends on its own effectiveness only. In the presence of synergy, however, the effectiveness of each medium depends not only on own effectiveness, but also on the spending level of the other medium. Consequently, as synergy increases, marginal spending on a medium increases at a rate proportional to the spending level of the *other* medium. Therefore, the optimal spending on the more effective medium increases slowly, while the optimal spending on the less effective medium increases rapidly. Hence, the proportion of budget allocated to the more effective medium decreases as synergy increases.

However the above results assume that sales evolve deterministically in response to multimedia advertising. Consequently, it is natural to question whether these results would hold in turbulent and volatile markets, where sales growth is uncertain. Uncertainty in sales response raises questions of significant interest. How does synergy influence the long-term profitability of the brand's IMC program under uncertainty? Does synergy affect the variability in the brand's long-term profitability? To address these issues, we next extend the IMC model to uncertain markets.

3. EXTENDED IMC MODEL

3.1 Incorporating Uncertainty

Consider the IMC model specified by the equation,

$$\frac{dS}{dt} = \beta_1 u(t) + \beta_2 v(t) + \kappa u(t)\, v(t) - (1-\lambda) S(t) \tag{1}$$

where S(t) denotes brand sales at time t, dS/dt is the instantaneous rate of change in sales (i.e., sales growth), u(t)and v(t) are advertising efforts on two different media (e.g., television and print), β_1 and β_2 are the effectiveness of the two media, κ measures the magnitude of synergy between the two media, and λ represents the usual carryover effect. See Naik and Raman (2003) for further details. Equation (1) proposes a two-fold influence of each medium on the sales rate: direct effects arising from media efforts (β_1, β_2), and an indirect effect driven by synergy (κ). Thus, when $\kappa > 0$, the combined sales impact of (u, v) exceeds the sum of the independent effects ($\beta_1 u + \beta_2 v$).

To incorporate uncertainty, we re-write equation (1) in the equivalent differential form, suppressing the variable "t" for notational clarity,

$$dS = \{\beta_1 u + \beta_2 v + \kappa u v - (1-\lambda) S\} dt\ , \tag{2}$$

and then introduce the continuous-time error process dW to obtain

$$dS = \{\beta_1 u + \beta_2 v + \kappa u v - (1-\lambda) S\} dt + \sigma dW\ . \tag{3}$$

In equation (3), the error term dW is the differential increment of W(t), a stochastic process known as Brownian motion (or alternatively as a Wiener process), and defined by the twin properties: (a) for every (t_1, t_2), $t_1 < t_2$, the increment $W(t_2) - W(t_1)$ has zero mean; and (b) for every (t_1, t_2), $t_1 < t_2$, the increment $W(t_2) - W(t_1)$ is normally distributed with variance equal to ($t_2 - t_1$). Hence, as in econometric models, the random error term dW follows the normal distribution with E[dW] = 0 and Var[dW] = σ^2dt (see Arnold 1974, p. 41, 45). This methodology finds precedence in Rao (1986) who also used a Brownian motion process to capture uncertainty in a continuous-time sales advertising model.

Equation (3) thus captures the effects of both IMC and uncertainty in a continuous-time framework. Next, we derive the optimal IMC program and evaluate its long-term profitability using stochastic control theory (see Raman 1990, Raman and Chatterjee 1995, Mantrala, Raman and Desiraju 1997).

3.2 Optimal IMC Program and Long-term Profitability

The advertiser's problem is to determine the dynamically optimal IMC program under synergy and uncertainty. Let u^* and v^* denote the optimal effort invested in the two media. The advertiser determines (u^*, v^*) by maximizing the expected discounted profit over an infinite horizon, namely,

$$E_s[\int_0^\infty e^{-\rho t}\Pi(S(t),u(t),v(t))dt], \qquad (4)$$

subject to sales evolution via the stochastic differential equation (3), where $E_s[\cdot]$ is the expectation conditional upon the initial sales S(0) = s, ρ denotes the discount rate, $\Pi(s, u, v) = ms - u^2 - v^2$ is the profit function, and m is the profit margin (see, e.g., Fruchter and Kalish 1998).

The optimal IMC program may be derived either by stochastic dynamic programming or the Lagrange method. Chow (1997) observes that the Lagrange method is often simpler to use, easier to interpret and more direct since it gives the minimum information needed to find optimal policies. However, in our case, the stochastic dynamic programming method is preferable for two fundamental reasons: (i) the Lagrange method yields only the *slope* of the maximum expected profit function (also known as the value function), but that would be inadequate for our purposes because information on just the slope is insufficient to analyze the long-term profitability of the IMC program, and (ii) the stochastic dynamic programming approach is well-suited to find closed-loop policies under uncertainty.

We implement stochastic dynamic programming through Hamilton-Jacobi-Bellman theory and derive:

$$u^*(\theta) = \underset{u,v}{\text{ArgMax}} \left[E_s \int_0^\infty e^{-\rho t}\Pi(S(t),u(S(t)),v(S(t)))dt \right], \qquad (5a)$$

$$v^*(\theta) = \underset{u,v}{\text{ArgMax}} \left[E_s \int_0^\infty e^{-\rho t}\Pi(S(t),u(S(t)),v(S(t)))dt \right], \qquad (5b)$$

where the vector $\theta = (\beta_1, \beta_2, \kappa, \lambda, \sigma, \rho, m)'$ contains all the model parameters. In equations (5a, b), the notation u(S(t)) and v(S(t)) signifies the closed-loop nature of strategies, while $u^*(\theta)$ and $v^*(\theta)$ remind us that the resulting optimal strategies depend on model parameters (e.g., magnitude of synergy, κ).

Next, we substitute the optimal strategies (u^*, v^*) in equation (4) to evaluate the profitability of the IMC program. Let J(s) denote the maximum profit attained when we use the optimal IMC strategies throughout the planning horizon, starting from an arbitrary sales level s. Then, the expected long-term profitability (ELP) is given by the limiting value

$$ELP = \lim_{t\to\infty} E[J(S(t))], \qquad (6)$$

where the expectation $E[\cdot]$ integrates over all possible sales levels S = s. Similarly, we evaluate the variability of long-term profitability (VLP) by

$$\mathrm{VLP} = \lim_{t \to \infty} \mathrm{Var}[J(S(t))]. \tag{7}$$

Finally, we use equations (5a), (5b), (6) and (7) to derive new results.

4. Normative Results

Here we present four new propositions, whose proofs we relegate to the Appendices in order to maintain continuity of exposition.

We note that the key results of Naik and Raman (2003) were derived for deterministic markets; that is, when $\sigma^2 = 0$. In contrast, proposition 1 generalizes those findings to uncertain markets where $\sigma^2 \neq 0$. We simply state the results here because we already discussed their implications and intuition in the literature review.

Proposition 1. *In uncertain markets, the total media budget increases as synergy increases. Furthermore, the proportion of budget allocated to the more (less) effective medium decreases (increases) as synergy increases.*

Proof. In Appendix A, we prove that, for every $\sigma^2 \neq 0$ (i.e., in the presence of uncertainty),

(i) $\dfrac{\partial}{\partial \kappa}(u^* + v^*) > 0$

(ii) $\dfrac{\partial}{\partial \kappa}\left(\dfrac{u^*}{v^*}\right) = \begin{cases} - & \text{if} \quad \beta_1 > \beta_2 \\ + & \text{if} \quad \beta_1 < \beta_2 \end{cases}.$

The next two propositions elucidate the bottom-line implications of IMC programs.

Proposition 2. *In uncertain markets, the expected value of long-term profitability (ELP) of the optimal IMC program increases as synergy increases.*

Proof. In Appendix B, we prove that $\dfrac{\partial\, \mathrm{ELP}}{\partial \kappa} > 0$ for every $\sigma^2 \neq 0$.

Proposition 3. *In uncertain markets, the variability of long-term profitability (VLP) of the optimal IMC program is unaffected by the magnitude of synergy.*

Proof. In Appendix C, we prove that $\dfrac{\partial\, \mathrm{VLP}}{\partial \kappa} = 0$ for every $\sigma^2 \neq 0$.

The pragmatic implication of Propositions 2 and 3 is that brand managers should adopt an IMC perspective to increase the brand's profitability. That is, they should think of marketing communications not as a set of independent activities, but rather as a set of interconnected activities with potential synergies. By integrating these activities to build synergies, they not only increase the expected profitability in the long run, but also keep profit variability unaltered. Thus synergy imposes no tradeoff between profitability and variability. Consequently, an IMC perspective raises profit but leaves its variability unaffected, and so it behooves managers to build synergies.

One approach for building synergies is to capitalize on the catalytic effects of synergy by using ancillary activities, as in the example of BMW's usage of product placement in Bond movies. We next define this catalysis concept, and state its managerial significance.

DEFINITION. *A marketing activity is a catalyst if it has negligible direct effect on sales, but exerts substantial synergies with other activities. For example, in equation (3), the activity v is a catalyst in the media mix (u, v) if the direct effect* $\beta_2 \approx 0$*, but the synergy* $\kappa \neq 0$*.*

PROPOSITION 4. *An advertiser should allocate a non-zero budget to the catalytic activity even if it is completely ineffective.*
PROOF. In Appendix D, we prove that $v^* \neq 0$ even if $\beta_2 = 0$.

This result demonstrates that advertisers must act differently when they adopt an IMC perspective. According to models of advertising that ignore synergy effects, an advertiser allocates the total budget to various media in proportion to their relative effectiveness (e.g., see proposition 1 in Naik and Raman 2003), and so the medium that is completely ineffective receives zero budget; i.e., when $\beta_2 = 0$, $v^* = 0$. In contrast, an advertiser who implements an IMC program benefits from not only the direct effects, but also the joint effects of various activities. Therefore, they should *not* eliminate spending on an ineffective medium if it enhances the effectiveness of other activities due to its catalytic presence. Next we present cases from industrial and pharmaceutical marketing to exemplify catalytic effects.[2]

When industrial marketers advertise in trade journals, they do not make purchasing managers directly buy the advertised products, but rather such advertising tends to enhance the effectiveness of sales calls by enhancing familiarity of the brand or company. Consider the following example from the pharmaceutical industry. The distribution of product samples or collateral

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[2] We thank Kash Rangan and Alvin Silk for discussions that led to these insights and the industrial marketing example.

materials to physicians does not increase the sales of prescription medicines, but it enhances the effectiveness of detailing efforts by sales representatives (Parsons and Vanden Abeele 1981). Indeed, many ancillary activities such as billboards, publicity, corporate advertising, event sponsorship, in-transit ads, merchandising, or product placement may not increase sales directly. Yet advertisers spend millions of dollars on them. Why? Because these activities are catalysts that enhance the effectiveness of primary activities (e.g., advertising or salesforce efforts) by strengthening brand knowledge in consumers' memory (Keller 1998, p. 257).

5. Discussion

Here we further extend the above analyses to duopoly markets and elaborate both the influence of uncertainty and the relative roles of commitment and feedback in media buying.

5.1 Role of competition

We express the IMC model (1) in terms of market share X(t), rather than sales, to obtain a duopoly IMC model[3],

$$dX = \{\beta_1 u_1 + \gamma_1 v_1 + \kappa_1 u_1 v_1 - \beta_2 u_2 - \gamma_2 v_2 - \kappa_2 u_2 v_2 - \delta(2X-1)\}dt + \sigma(X)dW, \tag{8}$$

where X is market share of brand 1, u_i and v_i are brand i's advertising decisions, i = 1 and 2, δ captures the rate of "churn" in a competitive market (Prasad and Sethi 2003), and $\sigma(X)dW$ is the stochastic component. Note that, with probability one, $X(t) \in (0,1)$, provided $|\beta_1 u_1 + \gamma_1 v_1 + \kappa_1 u_1 v_1 - \beta_2 u_2 - \gamma_2 v_2 - \kappa_2 u_2 v_2| < \delta$ and $\sigma(X) > 0$ for all X, $\sigma(0) = 0$, $\sigma(1) = 0$. In words, these mathematical conditions prevent the duopoly from collapsing into a monopoly.[4] Furthermore, because competition for brand share is a zero-sum game, we do not specify a separate

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[3] We thank an anonymous reviewer for suggesting this model structure.

[4] More specifically, Gikhman and Skorohod (1972, p. 158) show that the process $dX = \mu(X)dt + \sigma(X)dW$ will remain within (r_1, r_2) provided that $\mu(r_1) > 0$, $\mu(r_2) < 0$, $\sigma(X) > 0$, for $X(t) \in (r_1, r_2)$, $\sigma(r_1) = 0$, and $\sigma(r_2) = 0$. In our model, $\mu(r_1) = F_1 - F_2 + \delta < 0$, $\mu(r_2) = F_1 - F_2 - \delta > 0$, where $F_i = \beta_i u_i + \gamma_i v_i + \kappa_i u_i v_i$, i = 1, 2, $r_1 = 0$, and $r_2 = 1$. The conjunction of these two conditions gives $|F_1 - F_2| < \delta$. Following Sethi (1983), we can impose additional conditions to drive the infinitesimal variance to zero as X approaches the boundary of (0, 1), for example, $\sigma(X) = X(1-X)$.

equation for the second brand. Each brand then maximizes its expected discounted net profit, $E_{x_i}[\int_0^\infty e^{-\rho_i t}\{m_i q\, y_i - (u_i^2 + v_i^2)\}dt]$, where $y_1 = x_1$ and $y_2 = 1 - x_1$, taking into account the share dynamics in (8) and the best response of the other brand. In the above integrand, q denotes the industry sales, and ρ_i and m_i denote the discount rate and profit margin of brand i, respectively. To this end, we derive the Hamilton-Jacobi-Bellman equation for each brand, follow the same logic that led to equation (A1), consider the general polynomial $J_i(x) = \sum_{k=0}^{k=n} J_{ki}\, x_i^k$, and find that n = 1 provides the optimal solution. Thus, the optimal IMC program for the first brand is given by

$$u_1^* = \frac{qm_1\{2(2\delta+\rho_1)\beta_1 + qm_1\gamma_1\kappa_1\}}{4(2\delta+\rho_1)^2 - q^2 m_1^2 \kappa_1^2}\text{, and} \tag{9}$$

$$v_1^* = \frac{qm_1\{2(2\delta+\rho_1)\gamma_1 + qm_1\beta_1\kappa_1\}}{4(2\delta+\rho_1)^2 - q^2 m_1^2 \kappa_1^2}. \tag{10}$$

The optimal IMC mix for the other brand is obtained by replacing the subscript 1 by the subscript 2. Finally, we analyze the optimal strategies (u_1^*, v_1^*) and (u_2^*, v_2^*) as well as their corresponding impact on long-term profit via J_i, i = 1,2 to discover the generalization:

PROPOSITION 5. *Propositions 1 through 4 generalize to duopoly markets.*
PROOF. See Appendix E.

The closed-form expressions in (9) and (10) can be differentiated with respect to any specific parameter to gain further insights into asymmetry in a firm's structure and the effects of the rival's parameters. Recent studies furnish such results in the context of Lanchester model dynamics. Specifically, Prasad and Sethi (2003) study a two-player dynamic competition model, solve it explicitly, and evaluate the comparative static signs with respect to all the model parameters for both symmetric and asymmetric firms (see their Tables 1 and 2), but ignore multiple media and synergy between them. Complementing these results, Naik, Raman and Winer (2005) analyze the N-brand competitive advertising model with multiple controls, estimate the model parameters using market data, evaluate the comparative static signs with respect to interaction between advertising and promotion, and discuss the effects of rival's parameters (see their Table 4).

5.2 Influence of uncertainty

Propositions 1 through 4 and their generalization to duopoly markets via proposition 5 seem to imply that uncertainty plays no role. This interpretation is not accurate. Specifically, uncertainty does affect sales evolution *directly* via equations (3) and (8), making the level and growth of sales less predictable in the foreseeable future. Furthermore, uncertainty *directly* affects the variability in long-term profit (see equation C2), increasing downside risks of losses and bankruptcies. Thus, uncertainty has serious consequences on both the dimensions of sales and profit. Hence, the role of uncertainty should not be ignored in the analyses. When we explicitly include it in the analyses, we learn that managers should *not alter* their decisions by increasing or decreasing budget in response to the effects of uncertainty on sales and profit.

This finding is important because it sheds light on existing practice, which is based on conflicting views. Specifically, advertising and media agencies advocate that managers should increase spending in response to demand shocks such as recessions (see ABP 1993). An empirical analysis of national media spending (not presented here), however, indicates that managers reduce the media budget during recessions on average. In contrast, our normative analysis reveals, via Propositions 1 through 5, that managers should "*stick to the course of action in uncertain times*." Indeed, this counter-intuitive insight arises not only in the proposed IMC models, but in many other models whose value function is linear in the state variable (e.g., Prasad and Sethi 2003, Sorger 1989, Sethi 1983).[5]

In sum, the force of these propositions lies in recognizing the distinction that "no action" on budget changes does not imply managerial "inaction", the former requiring knowledge of optimal decision-making under uncertainty, the discipline not to tinker with marketing budgets in the short term, and the commitment to building brands over the long term.

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[5] In a recent insightful analysis, due to Prasad and Sethi (2003), we learn another counter-intuitive result that brands with smaller market share should spend *more aggressively* on advertising than larger brands. This finding is contrary to the conventional practice of some firms to maintain share-of-voice proportional to market share (which implies smaller brands should spend less aggressively). Thus, managers should re-consider the validity of their decision rules in ever-changing dynamic markets.

5.3 Commitment versus Feedback

We note that the optimal advertising strategy for our dynamic model does not depend on the level of sales. This fact raises two questions: (a) is the model structure realistic, and (b) do managers buy media without weekly sales feedback?

To address issue (a), we observe that our model structure is based on Nerlove-Arrow (NA) dynamics, which has been empirically validated in hundreds of research studies in marketing since Palda's (1964) dissertation. More recently, Bucklin and Gupta (1999, p. 262) surveyed real managers and found that the commonly used advertising model in industry is based on the Koyck model, which is a discrete-time version of NA dynamics. In addition, using Dockers brand data, Naik and Raman (2003) provided empirical validation for the advertising model based on NA dynamics. Thus, the model structure we have used in this paper enjoys both managerial usage and empirical support.

As for question (b), consider the institutional facts of buying network time, which is sold in two markets: the upfront market and the scatter market (Tellis 1998, p. 351, Belch and Belch 2004, p. 358). In the upfront market, an advertiser buys network time before the season begins and *commits up to a year in advance* without knowing week-by-week sales for the upcoming year. On the other hand, the scatter market is a "spot" market where network time can be purchased based on past sales performance. Consequently, constant or open-loop advertising strategies are implemented in upfront markets, whereas feedback or closed-loop strategies are implementable in the scatter markets only. It is important to recognize that, of the total network time sold, an overwhelming amount is bought in the upfront market relative to the scatter market. For example, approximately 80% of this year's network time is already *sold* in the upfront market (see Brough 2004) even before the year's Fall season began at the time of writing this manuscript and long before the Winter, Spring and Summer seasons that are due six to twelve months from now. Thus, substantial media buying relies on advance commitments, *without the use of feedback strategies*, and this feature of our model's prediction corroborates with the prevailing practice of advertisers as well as the workings of media institutions. In sum, our model based on NA dynamics has empirical and practical validity, and the resulting advertising strategy applies to the major proportion of an advertiser's spending decisions.

6. CONCLUDING REMARKS

We incorporated the roles of uncertainty and competition in an IMC model (Naik and Raman 2003), and solved the resulting stochastic control problem to determine the optimal IMC program. Our results elucidate the profit implications of IMC programs, explain the catalytic effects of synergy in IMC contexts, and generalize the findings of Naik and Raman (2003) to uncertain and competitive markets. Much remains to be done in the area of multimedia communications; for example, limited knowledge exists on planning the IMC budget and allocation during recessionary times. We hope that our research provides the impetus and methodology to investigate such managerially relevant topics.

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APPENDIX A: PROOF OF PROPOSITION 1

Here we solve the stochastic control problem of maximizing the objective functional in equation (4) subject to the equation (3), which explicitly recognizes the presence of uncertainty (since variance $\sigma^2 \neq 0$).

Let J(s) denote the maximum profit attained when we implement the optimal controls u(s) and v(s), starting from the initial sales level s. Then the Hamilton-Jacobi-Bellman (HJB) equation specifies the differential equation for J:

$$-\rho J + \underset{u,v}{\text{Max}}\,[\Pi(s,u,v) + J_s f(s,u,v) + 0.5\sigma^2 J_{ss}] = 0, \qquad \text{(A1)}$$

where $J_s = \partial J/\partial s$, $J_{ss} = \partial^2 J/\partial s^2$, $\Pi(s, u, v) = ms - u^2 - v^2$, and f(s, u, v) is the drift term in equation (3). We differentiate terms within the square brackets in (A1) with respect to the controls u and v, and solve the resulting first-order conditions by using the Cramer's rule to obtain:

$$u = \frac{J_s(2\beta_1 + \beta_2 \kappa J_s)}{4 - \kappa^2 J_s^{\,2}}, \qquad v = \frac{J_s(2\beta_2 + \beta_1 \kappa J_s)}{4 - \kappa^2 J_s^{\,2}}. \qquad \text{(A2)}$$

We note that the controls (u, v) in equation (A2) are closed-loop because they depend on the level of sales s via the marginal profit rate $J_s = \partial J/\partial s$, where J(s) is an unknown function of s (to be determined).

Next, by substituting u and v in the equation (A1), we get the second-order ordinary differential equation (ODE):

$$\begin{aligned} &-8ms - 4\sigma^2 J_{ss} + \sigma^2 \kappa^2 J_s^{\,2} J_{ss} + 8sJ_s(1-\lambda) + 8\rho J - 4\sigma^2 J_{ss} \\ &+ J_s^{\,2} J_{ss} \kappa^2 \sigma^2 - 2J_s^{\,2}\beta_1^{\,2} - 2J_s^{\,3}\kappa\beta_1\beta_2 - 2J_s^{\,2}\beta_2^{\,2} = 0 \end{aligned} \qquad \text{(A3)}$$

To solve this ODE analytically, we consider a polynomial solution $J(s) = \sum_{i=0}^{i=n} J_i s^i$, and apply the method of undetermined coefficients to find the set of coefficients $\{J_i\}$ and the order of polynomial, n. Specifically, we discover that n = 1 and so the resulting solution is

$$J(s) = J_0 + J_1 s, \qquad \text{(A4)}$$

where

$$J_0 = \frac{m(m(1-\lambda+\rho)\beta_1{}^2 + m^2\kappa\beta_1\beta_2 + m(1-\lambda+\rho)\beta_2{}^2)}{\rho(1-\lambda+\rho)(4(1-\lambda+\rho)^2 - m^2\kappa^2)}, \text{ and} \tag{A5}$$

$$J_1 = \frac{m}{1-\lambda+\rho}. \tag{A6}$$

Since $J_s = \partial J/\partial s = J_1$, we substitute (A6) in (A2) to obtain the optimal IMC strategies:

$$u^*(\theta) = \frac{m(\beta_2\kappa m + 2\beta_1(1-\lambda+\rho))}{4(1-\lambda+\rho)^2 - \kappa^2 m^2}, \quad v^*(\theta) = \frac{m(\beta_1\kappa m + 2\beta_2(1-\lambda+\rho))}{4(1-\lambda+\rho)^2 - \kappa^2 m^2}.$$
(A7)

Finally, we differentiate (A7) with respect to κ to get,

$$\frac{\partial u^*}{\partial \kappa} > 0 \text{ and } \frac{\partial v^*}{\partial \kappa} > 0, \tag{A8}$$

$$\frac{\partial}{\partial \kappa}\left(\frac{u^*}{v^*}\right) = \begin{cases} - & \text{if } \beta_1 > \beta_2 \\ + & \text{if } \beta_1 < \beta_2 \end{cases}. \tag{A9}$$

This completes the proof of proposition 1.

APPENDIX B: PROOF OF PROPOSITION 2

Here we prove proposition 2 by deriving the expression for expected long-term profitability (ELP). To this end, we first substitute s = S(t) in the equation (A4) of Appendix A, and evaluate the expectation,

$$E[J(S(t))] = J_0 + J_1\,\mu(t), \tag{B1}$$

where we denote $\mu(t) = E[S(t)]$. To find $E[S(t)]$, we obtain via equation (3),

$$\begin{aligned} E[dS] &= E[(\beta_1 u + \beta_2 v + \kappa u\, v - (1-\lambda)S)\,dt + \sigma dW] \\ &= (\beta_1 u + \beta_2 v + \kappa u\, v - (1-\lambda)\,E[S])dt + \sigma\, E[dW] \\ &= (\beta_1 u + \beta_2 v + \kappa u\, v - (1-\lambda)\,\mu)dt, \end{aligned} \tag{B2}$$

where the last equality follows from the property E[dW] = 0 (Arnold 1974, p. 41). In addition, it follows from Fubini's theorem (Harrison 1985, p. 131) that

$$E[dS] = d[E[S]] = d\mu . \tag{B3}$$

Therefore, from (B2) and (B3) together, we get

$$\frac{d\mu}{dt} = \beta_1 u + \beta_2 v + \kappa uv - (1-\lambda)\mu . \tag{B4}$$

We next implement the optimal controls u^* and v^* to obtain

$$\frac{d\mu}{dt} = C^* - (1-\lambda)\mu , \tag{B5}$$

where $C^* = \beta_1 u^* + \beta_2 v^* + \kappa u^* v^*$, and thus determine the optimal steady-state

$$\lim_{t\to\infty} \mu(t) = \frac{C^*}{1-\lambda} . \tag{B6}$$

We then evaluate the expected long-term profitability

$$\begin{aligned} ELP &= \lim_{t\to\infty} E[J(S(t))] \\ &= \lim_{t\to\infty} [J_0 + J_1 \mu(t)] \\ &= J_0 + J_1 \frac{C^*}{1-\lambda}, \end{aligned} \tag{B7}$$

where the second equality follows from (B1), the last equality follows from (B6), and J_0 and J_1 are given by the equations (A5) and (A6), respectively.

Finally, by differentiating (B7) with respect to synergy, we find that $\frac{\partial ELP}{\partial \kappa} > 0$ because both $\frac{\partial J_0}{\partial \kappa} > 0$ (using A5) and $\frac{\partial C^*}{\partial \kappa} > 0$ (using A8). This completes the proof of proposition 2.

APPENDIX C: PROOF OF PROPOSITION 3

We prove proposition 3 by evaluating the variance of long-term profitability:

$$\begin{aligned} \text{VLP} &= \lim_{t\to\infty} \text{Var}[J(S(t))] \\ &= \lim_{t\to\infty} \text{Var}[J_0 + J_1 S(t)] \\ &= \lim_{t\to\infty} J_1^2 \ \text{Var}[S(t)]. \end{aligned} \tag{C1}$$

To find Var[S(t)], we require the second moment of the sales process S(t). Hence, we apply Ito's Lemma to the stochastic process $y(t) = S(t)^2$, derive and solve the differential equation satisfied by $\mu_2(t) = E[y(t)] = E[S(t)^2]$, and determine Var[S(t)] through the relation $\text{Var}[S(t)] = E[S(t)^2] - (E[S(t)])^2 = \mu_2(t) - \mu(t)^2$. Thus, we obtain

$$\begin{aligned} \text{VLP} &= J_1^2 \ \{\lim_{t\to\infty} \text{Var}[S(t)]\} \\ &= \frac{m^2\,\sigma^2}{2\,(1-\lambda)\,(1-\lambda+\rho)^2} \end{aligned} \tag{C2}$$

It is clear from (C2) that $\dfrac{\partial \text{VLP}}{\partial \kappa} = 0$, which completes the proof.

APPENDIX D: PROOF OF PROPOSITION 4

Suppose that the activity v is completely ineffective so that $\beta_2 = 0$. Then, we substitute $\beta_2 = 0$ in equation (A7) to find that the optimal $v^* = \dfrac{\beta_1 \kappa m^2}{4(1+\rho-\lambda)^2 - \kappa^2 m^2}$, which is non-zero when $\kappa \neq 0$. This completes the proof.

APPENDIX E: PROOF OF PROPOSITION 5 (DUOPOLY)

We set up the Hamilton-Jacobi-Bellman equation for each of the two brands and follow the same logic that led to equation (A1). As before, we consider a polynomial solution $J(s) = \sum_{i=0}^{i=n} J_i\, s^i$, find that n = 1, and obtain the linear value function for each duopolist by solving for J_0 and J_1. Consequently, we determine the optimal IMC strategy of the first brand, which is given by equations (9) and (10). Using equations (9) and (10), we derive the comparative static results:

$$\frac{\partial u_1^*}{\partial \kappa_1} = \frac{q^2 m_1{}^2 \left(4\beta_1 q m_1 \kappa_1 (2\delta+\rho_1) + \gamma_1 \left(q^2 m_1{}^2 \kappa_1{}^2 + 4(2\delta+\rho_1)^2\right)\right)}{\left(q^2 m_1{}^2 \kappa_1{}^2 - 4(2\delta+\rho_1)^2\right)^2} > 0 \qquad \text{(E3)}$$

$$\frac{\partial v_1^*}{\partial \kappa_1} = \frac{q^2 m_1{}^2 \left(4\gamma_1 q m_1 \kappa_1 (2\delta+\rho_1) + \beta_1 \left(q^2 m_1{}^2 \kappa_1{}^2 + 4(2\delta+\rho_1)^2\right)\right)}{\left(q^2 m_1{}^2 \kappa_1{}^2 - 4(2\delta+\rho_1)^2\right)^2} > 0 \qquad \text{(E4)}$$

$$\frac{\partial}{\partial \kappa_1}\left(\frac{u_1^*}{v_1^*}\right) = -2\frac{\left(\beta_1{}^2 - \gamma_1{}^2\right)\left(q m_1 (2\delta+\rho_1)\right)}{\left(q m_1 \beta_1 \kappa_1 + 2\gamma_1 (2\delta+\rho_1)\right)^2}$$

(E5)

Thus, the expressions (E3)-(E5) establish that proposition 1 generalizes to the duopoly market.

Denoting the value function of duopolist k by J^k, k = 1, 2, we obtain the derivative of the first duopolist's value function $J^1(\theta)$ with respect to its synergy parameter κ_1,

$$\frac{\partial J^1}{\partial \kappa_1} = \frac{m_1{}^3 q^3 \{(2\beta_1{}^2 \kappa_1 m_1 q (2\delta+\rho_1) + 2\gamma_1{}^2 \kappa_1 m_1 q (2\delta+\rho_1) + \beta_1 \gamma_1 \left(q^2 m_1{}^2 \kappa_1{}^2 + 4(2\delta+\rho_1)^2\right)}{\rho_1 (2\delta+\rho_1)\{-\kappa_1{}^2 m_1{}^2 q^2 - 4(2\delta+\rho_1)^2\}^2}$$

. (E6)

Because (E6) is always positive, it establishes that proposition 2 holds in the duopoly case, following exactly the same reasoning as in Appendix B.

As in Appendix C, we apply Ito's Lemma to the stochastic process $Z(t) = X(t)^2$, derive and solve the differential equation satisfied by $\mu_2(t) = E[Z(t)] = E[X(t)^2]$, and determine Var[Z(t)] through the relation $\mathrm{Var}[Z(t)] = E[Z(t)^2] - (E[Z(t)])^2 = \mu_2(t) - \mu(t)^2$, thus obtaining:

$$VLP_1 = \frac{m_1{}^2 q^2 \sigma^2}{4\delta (2\delta+\rho_1)^2}. \qquad \text{(F7)}$$

Because $\frac{\partial VLP_1}{\partial \kappa_1} = 0$, we prove that proposition 3 holds for the duopoly case.

To prove that proposition 4 holds for the duopoly case, we substitute $\gamma_1 = 0$ in equation (10) to find that the optimal $v_1^* = \frac{(qm_1)^2 \beta_1 \kappa_1}{4(2\delta + \rho_1)^2 - q^2 m_1^{\ 2} \kappa_1^{\ 2}}$, which is non-zero when $\kappa_1 \neq 0$.

Finally, similar results can be obtained for the other brand by replacing the subscript 1 with 2 in the above expressions, which completes the proofs for proposition 5.